UNIT-I CONDUCTION

Heat transfer :-

Transmission of energy from one region to another region due to temp. difference.

Modes of Heat transfer

* Conduction
* Convection
* Radiation

Conduction :-

Heat is transfer from one region to another region with in a medium such as solid.

Convection :-

Heat is transfer between a solid surface and a fluid when they are at different temperatures is called as convection

Radiation :-

Heat is transfer from one body to another body without any intervening medium is known as radiation.

Example :-

Heat received by a man from fire place

Fourier law of conduction

Rate of heat conduction is proportional to the area measured normal to the ~~area~~ direction of heat flow and to the temp. gradient in that direction.

$$Q \propto -A \frac{dT}{dx}$$

$$Q = -KA \frac{dT}{dx}$$

where A – Area in m^2

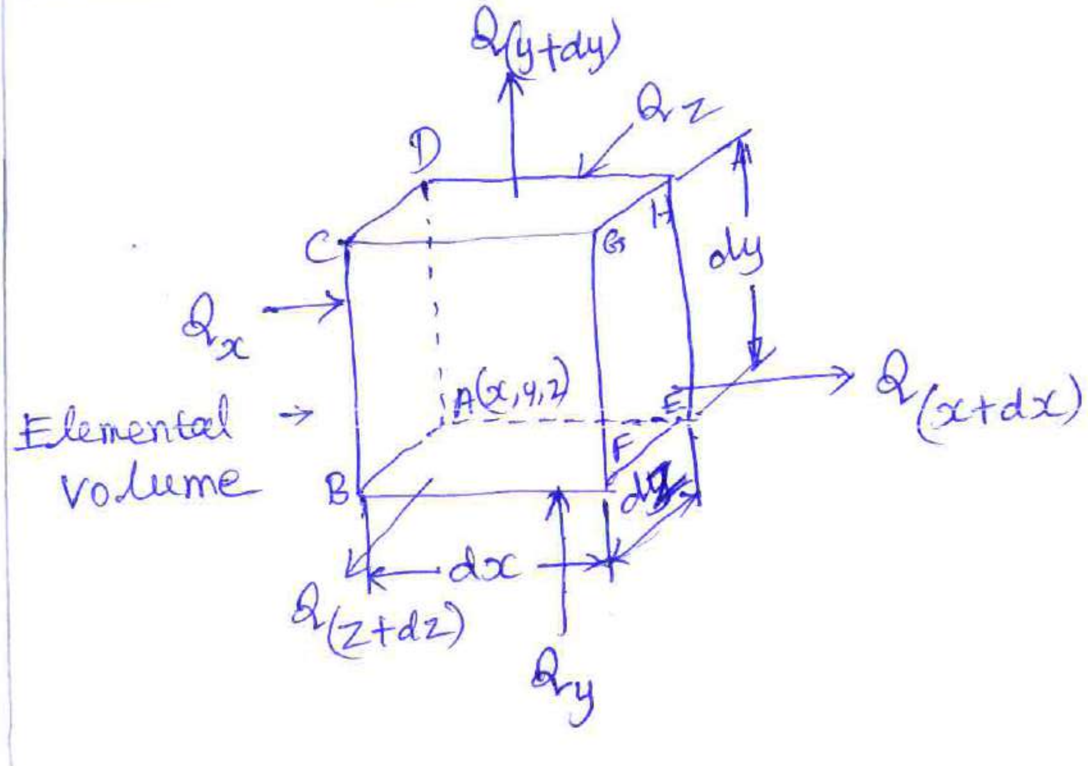
$\frac{dT}{dx}$ – Temp. gradient in K/m

K – Thermal conductivity in W/mK

Thermal conductivity

It is defined as the ability to conduct the heat.

General differential equation of heat conduction – cartesian co-ordinates



Consider a small cube element sides of dx, dy and dZ as shown in figure.

From 1st law of thermodynamics,

$$\left.\begin{array}{l}\text{Net heat conducted}\\ \text{into the element}\\ \text{from all the}\\ \text{co-ordinate directions}\end{array}\right\} + \left.\begin{array}{l}\text{Heat generated}\\ \text{with in the}\\ \text{element}\end{array}\right\} = \begin{array}{l}\text{Heat stored}\\ \text{in the}\\ \text{element}\end{array} \quad \text{(A)}$$

Let q_x - Heat flux in a direction of face ABCD

q_{x+dx} - Heat flux in a direction of face EFGH

The rate of heat flow into the element in 'x' direction through the face ABCD is

$$Q_x = q_x \cdot dy \cdot dZ$$

$$Q_x = -K_x \frac{\partial T}{\partial x} dy \cdot dZ \quad \text{——— (1)}$$

Where K - Thermal conductivity, w/mK

$\frac{\partial T}{\partial x}$ - Temp. gradient

The rate of heat flow out of the element in 'x' direction through the face EFGH is

$$Q_{x+dx} = Q_x + \frac{\partial}{\partial x}(Q_x) \cdot dx$$

$$= -K_x \cdot \frac{\partial T}{\partial x} \cdot dy \cdot dz + \frac{\partial}{\partial x}\left[-K_x \cdot \frac{\partial T}{\partial x} \cdot dy \cdot dz\right] \cdot dx$$

$$Q_{x+dx} = -K_x \frac{\partial T}{\partial x} \cdot dy \cdot dz - \frac{\partial}{\partial x}\left[K_x \cdot \frac{\partial T}{\partial x}\right] dx \cdot dy \cdot dz \quad \text{— (2)}$$

subtract the eqns (1) & (2)

(1) - (2), we get

$$Q_x - Q_{(x+dx)} = -K_x \cdot \frac{\partial T}{\partial x} \cdot dy \cdot dz - \left[-K_x \cdot \frac{\partial T}{\partial x} \cdot dy\, dz - \frac{\partial}{\partial x}\left[K_x \cdot \frac{\partial T}{\partial x}\right] dx \cdot dy \cdot dz \right]$$

$$= \cancel{-K_x \cdot \frac{\partial T}{\partial x} \cdot dy\, dz} + \cancel{K_x \cdot \frac{\partial T}{\partial x} \cdot dy \cdot dz} + \frac{\partial}{\partial x} \cdot \left[K_x \cdot \frac{\partial T}{\partial x}\right] dx \cdot dy \cdot dz$$

$$\therefore Q_x - Q_{(x+dx)} = \frac{\partial}{\partial x}\left[K_x \cdot \frac{\partial T}{\partial x}\right] \cdot dx \cdot dy \cdot dz \quad \text{— (3)}$$

similarly, we can write

$$Q_y - Q_{(y+dy)} = \frac{\partial}{\partial y}\left[K_y \cdot \frac{\partial T}{\partial y}\right] \cdot dx\, dy\, dz \quad \text{— (4)}$$

$$Q_z - Q_{(z+dz)} = \frac{\partial}{\partial z}\left[K_z \frac{\partial T}{\partial z}\right] \cdot dx\, dy\, dz \quad \text{— (5)}$$

Add the eqns (3), (4) & (5), we get

$$\left.\begin{array}{l}\text{Net heat}\\ \text{conducted}\end{array}\right\} = \left\{ \frac{\partial}{\partial x}\left[K_x \cdot \frac{\partial T}{\partial x}\right] + \frac{\partial}{\partial y}\left[K_y \cdot \frac{\partial T}{\partial y}\right] + \frac{\partial}{\partial z}\left[K_z \cdot \frac{\partial T}{\partial z}\right] \right\} \cdot dx \cdot dy \cdot dz \quad \text{— (6)}$$

$$\left.\begin{array}{l}\text{Heat generated}\\ \text{within the element}\end{array}\right\} Q = \dot{q}\, dx \cdot dy \cdot dz \quad \text{— (7)}$$

$$\left.\begin{array}{l}\text{Heat stored}\\ \text{in the element}\end{array}\right\} = \left.\begin{array}{c}\text{Mass of}\\ \text{the}\\ \text{element}\end{array}\right\} \times \left.\begin{array}{c}\text{Specific}\\ \text{heat of}\\ \text{the element}\end{array}\right\} \times \left.\begin{array}{c}\text{Rise in}\\ \text{temp. of}\\ \text{element}\end{array}\right]$$

$$= m \times C_p \times \frac{\partial T}{\partial t}$$

$$= \rho \times dx \cdot dy \cdot dz \times C_p \times \frac{\partial T}{\partial t} \quad \text{— (8)}$$

[$\because$ Mass = Density $\times$ volume
Density $= \rho$
Volume $= dx \cdot dy \cdot dz$]

Substituting the eqns (6), (7) & (8) in eqn (A)

$$\left\{\frac{\partial}{\partial x}\left[K_x \cdot \frac{\partial T}{\partial x}\right] + \frac{\partial}{\partial y}\left[K_y \cdot \frac{\partial T}{\partial y}\right] + \frac{\partial}{\partial z}\left[K_z \cdot \frac{\partial T}{\partial z}\right]\right\} \cdot dx \cdot dy \cdot dz$$

$$+ \dot{q} \cdot dx \cdot dy \cdot dz = \rho C_p \cdot \frac{\partial T}{\partial t} \cdot dx \cdot dy \cdot dz$$

$$\frac{\partial}{\partial x}\left[K_x \cdot \frac{\partial T}{\partial x}\right] + \frac{\partial}{\partial y}\left[K_y \cdot \frac{\partial T}{\partial y}\right] + \frac{\partial}{\partial z} \cdot \left[K_z \cdot \frac{\partial T}{\partial z}\right] + \dot{q} = \rho C_p \cdot \frac{\partial T}{\partial t}$$

Considering the material is isotropic.

$$\therefore \quad K_x = K_y = K_z = K = \text{constant}$$

$$K\left\{\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right] + \dot{q} = \rho C_p \cdot \frac{\partial T}{\partial t}$$

Divided by K on both sides

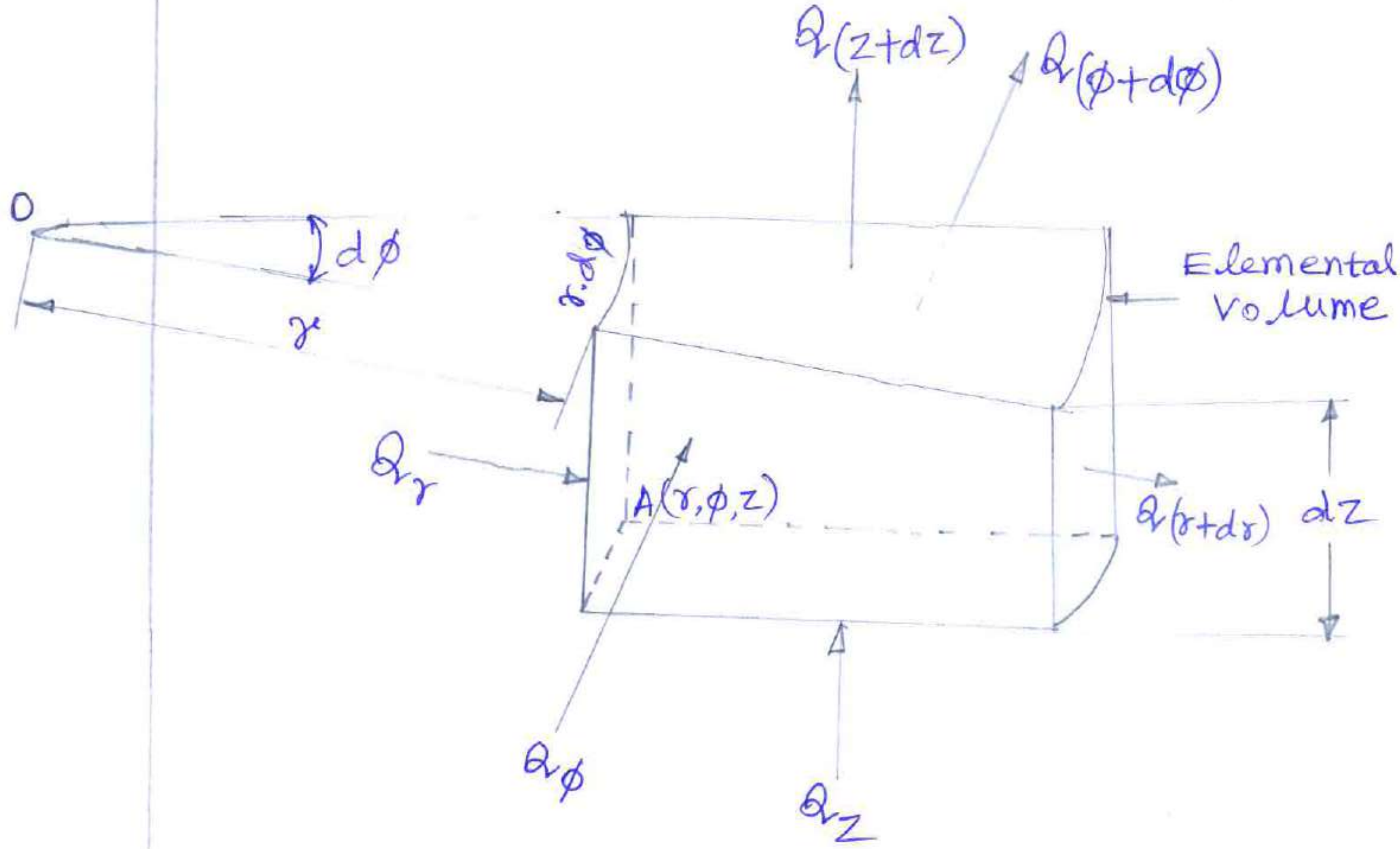
$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{K} = \frac{\rho C_p}{K} \cdot \frac{\partial T}{\partial t}$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{K} = \frac{1}{\alpha} \cdot \frac{\partial T}{\partial t}$$

$$\left[\because \alpha = \frac{K}{\rho . C_p} \text{ in } m^2/s\right]$$

It is a general 3D heat conduction equation in Cartesian co-ordinates.

General differential equation of heat conduction - cylindrical co-ordinates



Consider a small cylindrical element of sides dr, $r.d\phi$ and dZ as shown in figure.

Volume of the element, $dV = r.d\phi.dr.dZ$

Let. assume that K, C_p and ρ are constant.

From 1st law of thermodynamics,

$$\left.\begin{array}{l}\text{Net heat conducted}\\ \text{into the element}\\ \text{from all the}\\ \text{co-ordinate directions}\end{array}\right\} + \left.\begin{array}{l}\text{Heat generated}\\ \text{with in the}\\ \text{element}\end{array}\right\} = \left.\begin{array}{l}\text{Heat Stored}\\ \text{in the}\\ \text{element}\end{array}\right\} \quad (1)$$

Net heat conducted into element from all the co-ordinate directions

Heat entering in the element through (r, ϕ) plane in time $d\theta$ (z direction)

From Fourier law of heat conduction

$$Q_z = -K\,(r.d\phi.dr)\frac{\partial T}{\partial z}\,d\theta \qquad \left[\because Q = -KA\frac{dT}{dx}\right]$$

Heat leaving from the element through (r, ϕ) plane in time $d\theta$ (z direction)

$$Q_{z+dz} = Q_z + \frac{\partial}{\partial z}(Q_z)\,dz$$

Net heat conducted into the element through (r, ϕ) plane in time $d\theta$ (z direction)

$$Q_z - Q_{z+dz} = -\frac{\partial}{\partial z}(Q_z)\,dz$$

$$= \frac{\partial}{\partial z}\left[K\,(r.d\phi.dr)\,\frac{\partial T}{\partial z}.d\theta\right]dz$$

$$= K\,\frac{\partial^2 T}{\partial z^2}(dr.\,rd\phi.dz)\,d\theta \quad \text{— } ②$$

Heat entering in the element through (ϕ, z) plane in time $d\theta$ (r direction)

$$Q_r = -K\,(r.d\phi.dz)\frac{\partial T}{\partial r}.d\theta$$

Heat leaving from the element through (ϕ, z) plane in time $d\theta$, $Q_{r+dr} = Q_r + \frac{\partial}{\partial r}(Q_r).dr$

Net heat conducted

$$Q_r - Q_{r+dr} = -\frac{\partial}{\partial r}(Q_r).dr$$

$$= -\frac{\partial}{\partial r}\left[-K\,(r.d\phi.dz).\,\frac{\partial T}{\partial r}.d\theta\right].dr$$

$$= K.(dr.d\phi.dz)\frac{\partial}{\partial r}\left[r.\frac{\partial T}{\partial r}\right].d\theta$$

$$= K\,(dr.d\phi.dz)\,r\left[\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}\right].d\theta$$

Net heat conducted through (ϕ, z) Plane

$$= K(dr.\,rd\phi.dz)\left[\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}\right].d\theta \quad — ③$$

Heat entering in the element through (z, r) Plane in time $d\theta$ (ϕ direction)

$$Q_\phi = -K\,(dr.dz)\frac{\partial T}{r.\partial\phi}\cdot d\theta$$

Heat leaving from the element through (z, r) plane in time $d\theta$

$$Q_{\phi+d\phi} = Q_\phi + \frac{\partial}{r.\partial\phi}(Q_\phi)\cdot r\,d\phi$$

Net heat conducted into the element through (z, r) plane in time $d\theta$

$$Q_\phi - Q_{\phi+d\phi} = -\frac{\partial}{r.\partial\phi}(Q_\phi)\,r.\,d\phi$$

$$= -\frac{\partial}{\cancel{r}.\partial\phi}\left[-K(dr.dz).\frac{\partial T}{r\partial\phi}.d\theta\right]\cancel{r}.d\phi$$

$$= K\frac{\partial}{\partial\phi}\left[\frac{1}{r}\frac{\partial T}{\partial\phi}\right](dr.d\phi.dz).d\theta \quad — ④$$

Net heat conducted into element from all the co-ordinate directions,

Add the eqns ②, ③ & ④, we get

$$= K \cdot \frac{\partial^2 T}{\partial z^2}(dr \cdot r d\phi \cdot dz)\, d\theta +$$

$$K(dr \cdot r d\phi \cdot dz)\left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right] d\theta +$$

$$K\left[\frac{1}{r^2} \cdot \frac{\partial^2 T}{\partial \phi^2}\right](dr \cdot r d\phi \cdot dz)\, d\theta$$

$$= K(dr \cdot r d\phi \cdot dz)\, d\theta \left[\frac{\partial^2 T}{\partial z^2} + \frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2}\right]$$

$$= K(dr \cdot r d\phi \cdot dz) \cdot d\theta \left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2}\right] \quad (5)$$

Heat generated within the element

$$Q = \dot{q}(dr \cdot r d\phi \cdot dz) \cdot d\theta \quad (6)$$

Heat stored in the element

The increase in internal energy of the element is equal to the net heat stored in the element

$$\left.\begin{matrix}\text{Increase in internal} \\ \text{energy}\end{matrix}\right\} = \rho (dr \cdot r d\phi \cdot dz) \cdot c_p \cdot \frac{\partial T}{\partial \theta} \times d\theta \quad (7)$$

Substitute the eqns (5), (6) & (7) in eqn (1), we get

$$K(dr \cdot r d\phi \cdot dz) \cdot d\theta \left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2}\right] +$$

$$\dot{q}(dr \cdot r d\phi \cdot dz) \cdot d\theta = \rho \cdot (dr \cdot r d\phi \cdot dz) \cdot c_p \cdot \frac{\partial T}{\partial \theta} \times d\theta$$

Divided by $(dr \cdot r d\phi \cdot dz) \cdot d\theta$ on both sides,

$$K\left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2}\right] + \dot{q} = \rho \cdot c_p \cdot \frac{\partial T}{\partial \theta}$$

Divided by 'K' on both sides.

$$\left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2}\right] + \frac{\dot{q}}{K} = \frac{\rho C_p}{K} \cdot \frac{\partial T}{\partial \theta}$$

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{K} = \frac{1}{\alpha} \cdot \frac{\partial T}{\partial \theta}$$

$$\left[\because \alpha = \frac{K}{\rho C_p}\right]$$

It is a general 3D heat conduction equation in cylindrical (Polar) co-ordinates.

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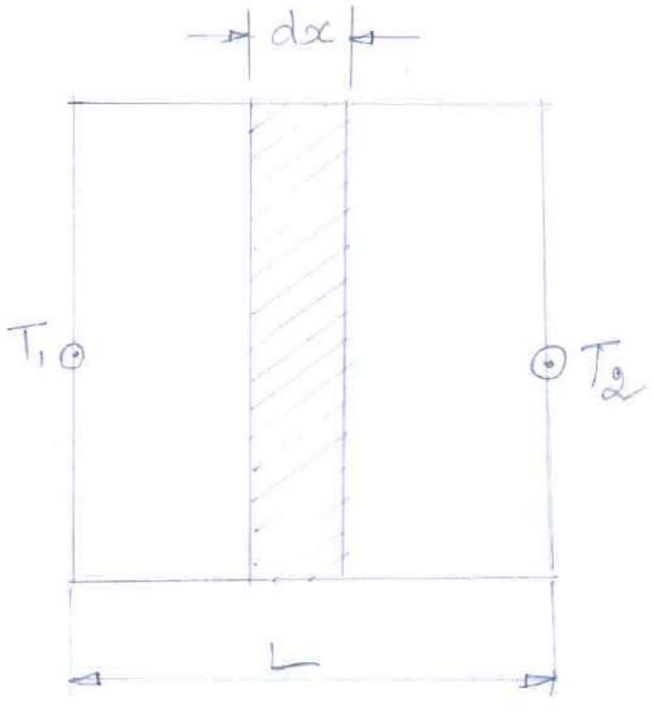
1 Dimensional Steady State heat conduction

Conduction through Plane wall



Consider a plane wall of uniform thermal conductivity K, thickness L with inner temp. T_1 and Outer temp. T_2

Let us consider a small elemental area of thickness 'dx'

From Fouriers law of conduction

$$Q = -KA \cdot \frac{dT}{dx}$$

$$Q \cdot dx = -KA \cdot dT$$

Integrating the above eqn b/w the limits of 0 to L & T_1 to T_2.

$$\int_0^L Q \cdot dx = -\int_{T_1}^{T_2} K \cdot A \, dT$$

$$Q \int_0^L dx = -KA \int_{T_1}^{T_2} dT$$

$$Q \left[x\right]_0^L = -K \cdot A \left[T\right]_{T_1}^{T_2}$$

$$Q \left[L - 0\right] = -KA \left[T_2 - T_1\right]$$

$$Q \times L = KA \left[T_1 - T_2\right]$$

$$Q = \frac{KA}{L} \left[T_1 - T_2\right] \quad \text{—— (1)}$$

$$Q = \frac{T_1 - T_2}{\frac{L}{KA}}$$

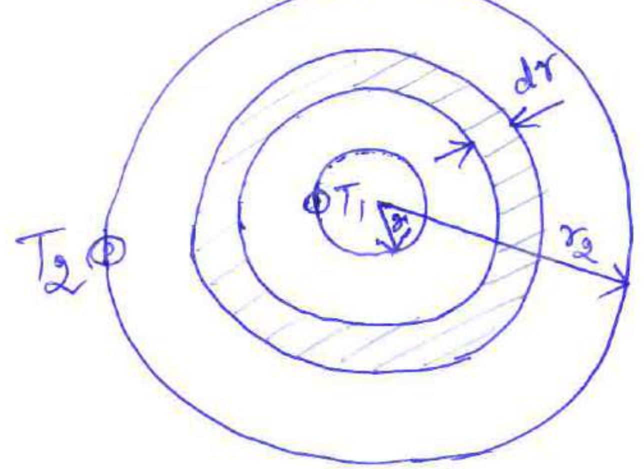
$$Q = \frac{\Delta T_{overall}}{R}$$

[$\because$ $\Delta T = T_1 - T_2$

$R = \frac{L}{KA}$

R = Thermal Resistance]

Conduction through hollow cylinder



Consider a hollow cylinder of inner radius r_1, outer radius r_2, inner temp. T_1, outer temp. T_2 and Thermal conductivity K.

Let us consider a small elemental area of thickness 'dr'

From Fourier law of conduction

W.K.T. $Q = -KA\frac{dT}{dr}$

Area of cylinder $= 2\pi rL$

$\therefore Q = -K.2\pi rL.\frac{dT}{dr}$

$Q \times \frac{dr}{r} = -K.2\pi L.dT$

Integrating the above eqn from r_1 to r_2 and T_1 to T_2

$$Q \int_{r_1}^{r_2} \frac{dr}{r} = -K 2\pi L \int_{T_1}^{T_2} dT \qquad \left[\because \int \frac{1}{x} \cdot dx = \ln x\right]$$

$$Q \left[\ln r\right]_{r_1}^{r_2} = -K 2\pi L \left[T\right]_{T_1}^{T_2}$$

$$Q \left[\ln r_2 - \ln r_1\right] = -K 2\pi L \left[T_2 - T_1\right]$$

$$Q \ln\left[\frac{r_2}{r_1}\right] = 2\pi L K \left[T_1 - T_2\right]$$

$$Q = \frac{2\pi L K \left[T_1 - T_2\right]}{\ln\left[\frac{r_2}{r_1}\right]}$$

$$Q = \frac{T_1 - T_2}{\frac{1}{2\pi L K} \ln\left[\frac{r_2}{r_1}\right]}$$

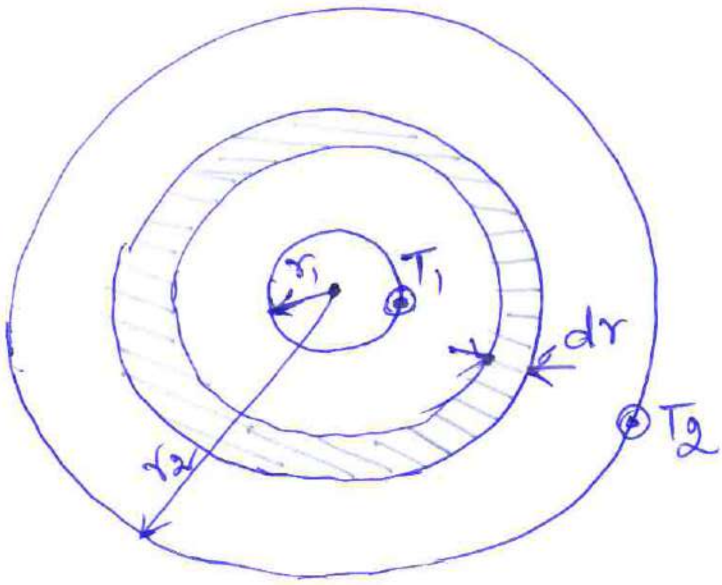
$$Q = \frac{\Delta T_{overall}}{R}$$

where, $\Delta T = T_1 - T_2$

$$R = \frac{1}{2\pi L K} \ln\left[\frac{r_2}{r_1}\right]$$

R = Thermal resistance of hollow cylinder

Conduction through hollow sphere



Consider a hollow sphere of inner radius r_1, outer radius r_2, inner temp. T_1, outer temp. T_2 and thermal conductivity K.

Let us consider a small elemental area of thickness 'dr'

From Fourier law of heat conduction,

$$Q = -KA \cdot \frac{dT}{dr}$$

Area of sphere $= 4\pi r^2$

$$Q = -K \cdot 4\pi r^2 \cdot \frac{dT}{dr} \quad -①$$

$$Q \times \frac{dr}{r^2} = -K \cdot 4\pi \cdot dT$$

Integrating on both sides, we get

$$Q \int_{r_1}^{r_2} \frac{dr}{r^2} = -\int_{T_1}^{T_2} 4\pi \cdot K \cdot dT$$

$$Q \int_{r_1}^{r_2} \frac{dr}{r^2} = -4\pi K \int_{T_1}^{T_2} dT$$

$$\int r^{-2} \cdot dr$$

$$Q \left[\frac{-1}{r} \right]_{r_1}^{r_2} = -4\pi K \left[T \right]_{T_1}^{T_2}$$

$$\frac{r^{-2+1}}{-2+1}$$

$$Q \left[-\frac{1}{r_2} - \left(-\frac{1}{r_1} \right) \right] = -4\pi K \left[T_2 - T_1 \right]$$

$$\frac{r^{-1}}{-1} = -\frac{1}{r}$$

$$Q \left[\frac{1}{r_1} - \frac{1}{r_2} \right] = 4\pi K \left[T_1 - T_2 \right]$$

$$\int x^n \cdot dx = \frac{x^{n+1}}{n+1}$$

$$Q \left[\frac{r_2 - r_1}{r_1 \cdot r_2} \right] = 4\pi K \left[T_1 - T_2 \right]$$

$$Q = \frac{4\pi K \left[T_1 - T_2 \right]}{\left[\frac{r_2 - r_1}{r_1 \cdot r_2} \right]}$$

$$Q = \frac{T_1 - T_2}{\left[\frac{r_2 - r_1}{4\pi K \cdot r_1 \cdot r_2} \right]}$$

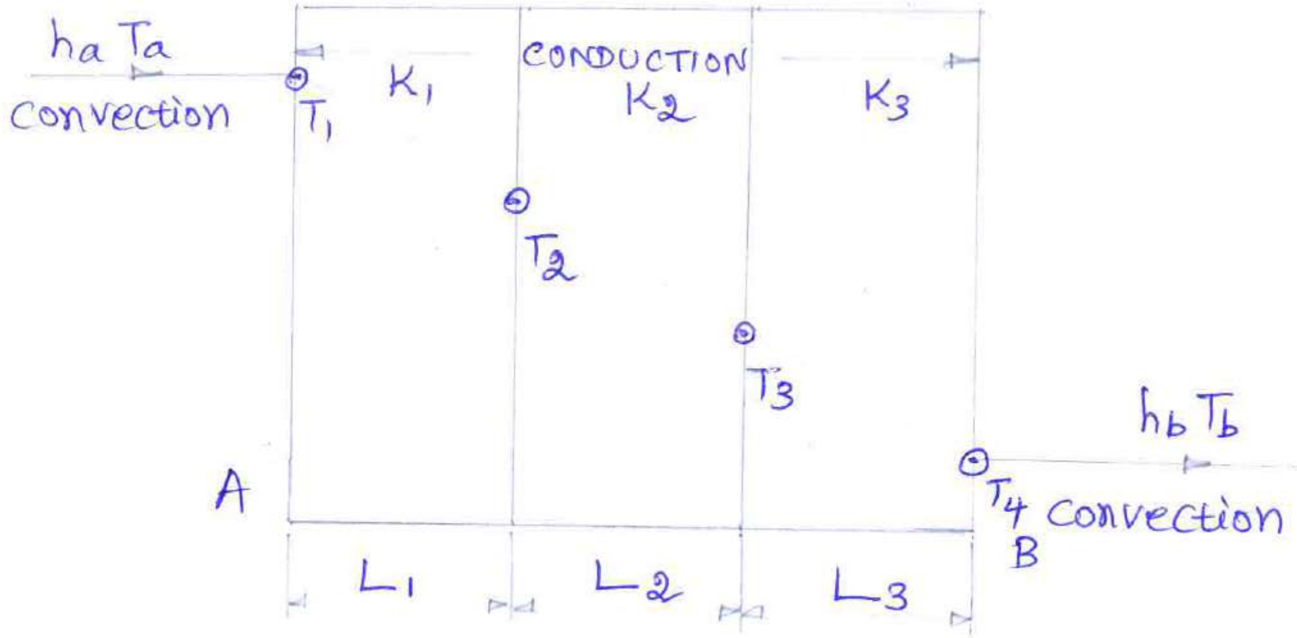
$$Q = \frac{\Delta T_{overall}}{R}$$

where, $\Delta T = T_1 - T_2$

$$R = \frac{r_2 - r_1}{4\pi K \cdot r_1 \cdot r_2}$$

R = Thermal resistance of hollow sphere

Heat transfer through a composite Plane wall



Consider a composite wall of thickness L_1, L_2 and L_3 having thermal conductivity K_1, K_2 and K_3 respectively.

It is assumed that the interior and exterior Surface of the system are Subjected to convection at mean temperatures T_a and T_b with H.T. coefficient h_a and h_b respectively.

Inside the composite wall, the Slabs are subjected to conduction

From Newton's law of cooling,

Heat Transfer by convection at side A is,

$$Q = h_a . A[T_a - T_1] \quad \text{——— (1)}$$

Heat transfer by conduction at slab ① is,

$$Q = \frac{K_1 A[T_1 - T_2]}{L_1} \quad \text{— (2)}$$

Similarly at slab ② & slab ③ is

$$Q = \frac{K_2 A[T_2 - T_3]}{L_2} \quad \text{— (3)}$$

$$Q = \frac{K_3 A[T_3 - T_4]}{L_3} \quad \text{— (4)}$$

Heat Transfer by convection at side B is,

$$Q = h_b . A[T_4 - T_b] \quad \text{— (5)}$$

w.k.T.,

$$T_a - T_1 = Q \times \frac{1}{h_a . A} \quad \text{— (6)}$$

$$T_1 - T_2 = Q \times \frac{L_1}{K_1 A} \quad \text{— (7)}$$

$$T_2 - T_3 = Q \times \frac{L_2}{K_2 A} \quad \text{— (8)}$$

$$T_3 - T_4 = Q \times \frac{L_3}{K_3 A} \quad \text{— (9)}$$

$$T_4 - T_b = Q \times \frac{1}{h_b . A} \quad \text{— (10)}$$

Add the eqns ⑥, ⑦, ⑧, ⑨ & ⑩ on both sides, we get

$$T_a - T_b = Q\left[\frac{1}{h_a \cdot A} + \frac{L_1}{K_1 \cdot A} + \frac{L_2}{K_2 \cdot A} + \frac{L_3}{K_3 \cdot A} + \frac{1}{h_b \cdot A}\right]$$

$$Q = \frac{T_a - T_b}{\left[\frac{1}{h_a \cdot A} + \frac{L_1}{K_1 \cdot A} + \frac{L_2}{K_2 \cdot A} + \frac{L_3}{K_3 \cdot A} + \frac{1}{h_b \cdot A}\right]}$$

$$Q = \frac{\Delta T_{overall}}{R} \quad \text{--- (11)}$$

where, $\Delta T = T_a - T_b$

Thermal Resistance, $R = R_a + R_1 + R_2 + R_3 + R_b$

$$R = \frac{1}{h_a \cdot A} + \frac{L_1}{K_1 A} + \frac{L_2}{K_2 A} + \frac{L_3}{K_3 A} + \frac{1}{h_b \cdot A}$$

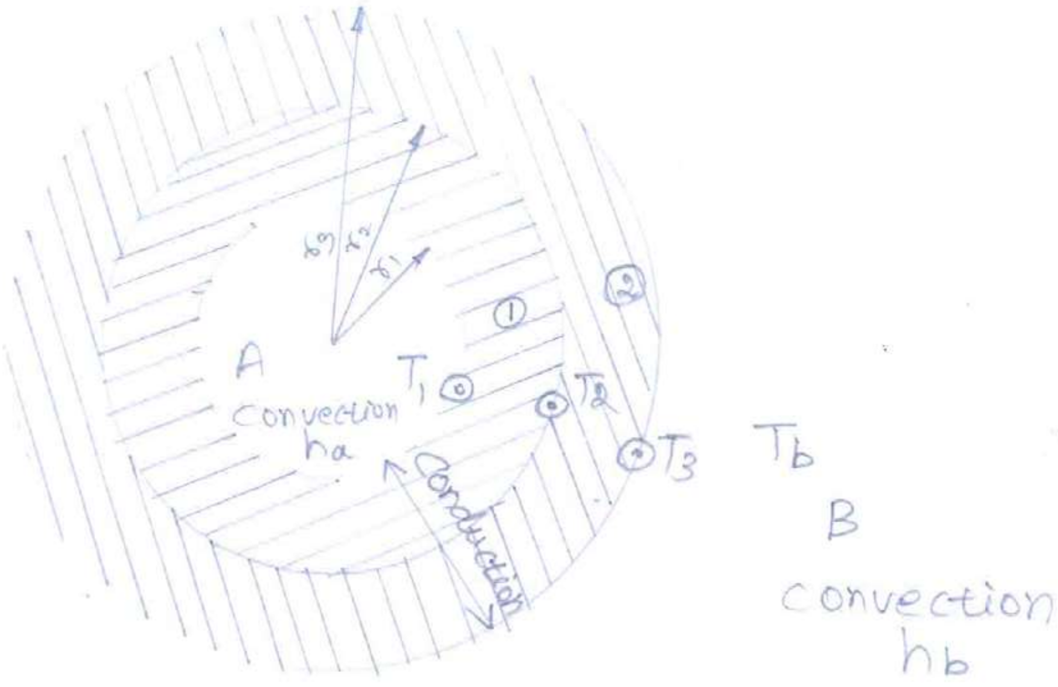
w.k.T. $R = \frac{1}{UA}$

$$Q = \frac{T_a - T_b}{\frac{1}{UA}}$$

$$Q = UA(T_a - T_b)$$

where, U – overall heat transfer co-efficient in W/m^2K

Heat transfer through composite pipes or cylinders



A hot fluid at a temperature T_a with heat transfer co-efficient ha, flowing through a pip is separated by two layers from atmosphere as shown in figure.

Let the thermal conductivities be K_1 & K_2.

outside surface heat is being ~~transferred~~ to a cold fluid at a temp. T_b with heat transfer co-efficient h_b.

Heat transfer by convection at side A is,

$$Q = h_a \cdot A[T_a - T_1]$$

Here Inner Area, $A = 2\pi r_1 L$

$$Q = 2\pi r_1 L\, h_a[T_a - T_1] \quad \text{—— (1)}$$

Heat transfer by conduction at section ① is,

$$Q = \frac{2\pi L K_1 [T_1 - T_2]}{\ln\left[\frac{r_2}{r_1}\right]} \quad \text{—— ②} \quad \text{[Eqn from hollow cylinder]}$$

similarly, Heat transfer by conduction at section ② is,

$$Q = \frac{2\pi L K_2 [T_2 - T_3]}{\ln\left[\frac{r_3}{r_2}\right]} \quad \text{—— ③}$$

Heat transfer by convection at side B is,

$$Q = h_b \cdot A [T_3 - T_b]$$

Here outer Area, $A = 2\pi r_3 \cdot L$

$$Q = 2\pi r_3 L h_b [T_3 - T_b] \quad \text{—— ④}$$

W.K.T.

$$T_a - T_1 = \frac{Q}{2\pi L r_1 h_a} \quad \text{—— ⑤}$$

$$T_1 - T_2 = \frac{Q}{2\pi L K_1} \ln\left(\frac{r_2}{r_1}\right) \quad \text{—— ⑥}$$

$$T_2 - T_3 = \frac{Q}{2\pi L K_2} \ln\left(\frac{r_3}{r_2}\right) \quad \text{—— ⑦}$$

$$T_3 - T_b = \frac{Q}{2\pi r_3 L h_b} \quad \text{—— ⑧}$$

Add the eqns ⑤, ⑥, ⑦ & ⑧ on both sides,

$$T_a - T_b = \frac{Q}{2\pi L}\left[\frac{1}{h_a r_1} + \frac{\ln(r_2/r_1)}{K_1} + \frac{\ln(r_3/r_2)}{K_2} + \frac{1}{h_b r_3}\right]$$

$$Q = \frac{2\pi L\left[T_a - T_b\right]}{\frac{1}{h_a r_1} + \frac{\ln(r_2/r_1)}{K_1} + \frac{\ln(r_3/r_2)}{K_2} + \frac{1}{h_b r_3}}$$

$$Q = \frac{T_a - T_b}{\frac{1}{2\pi L}\left[\frac{1}{h_a r_1} + \frac{\ln(r_2/r_1)}{K_1} + \frac{\ln(r_3/r_2)}{K_2} + \frac{1}{h_b r_3}\right]}$$

$$Q = \frac{\Delta T_{overall}}{R} \quad \text{—— (9)}$$

where, $\Delta T = T_a - T_b$

$$R = \frac{1}{2\pi L}\left[\frac{1}{h_a r_1} + \frac{\ln(r_2/r_1)}{K_1} + \frac{\ln(r_3/r_2)}{K_2} + \frac{1}{h_b r_3}\right]$$

W.K.T. $R = \frac{1}{UA}$

$$Q = \frac{T_a - T_b}{\frac{1}{UA}}$$

$$Q = UA.(T_a - T_b) \quad \text{—— (10)}$$

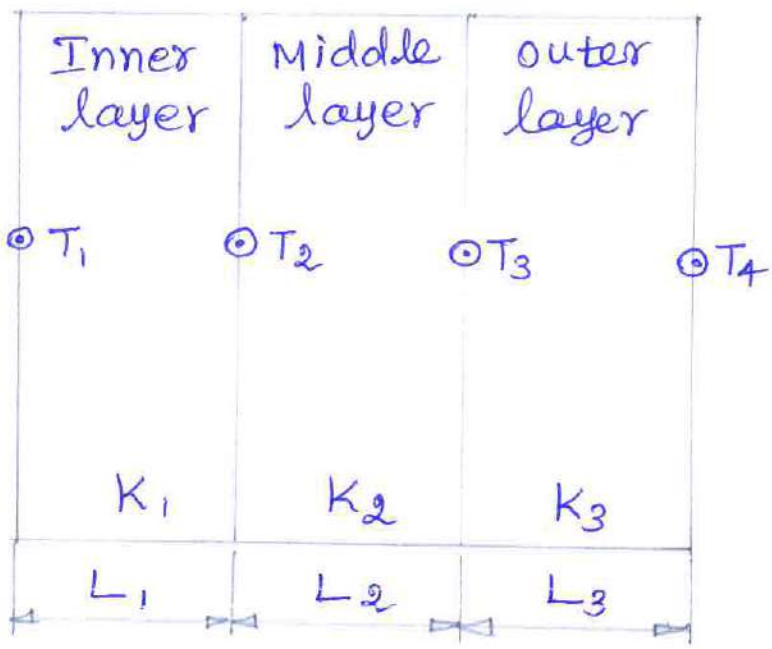
where, U = overall heat transfer coefficient, W/m^2K

A = Area = $2\pi r_3 L$

Problem:-

1. A furnace wall is made up of three layers, inside layer with thermal conductivity 8.5 W/mK, the middle layer with conductivity 0.25 W/mK, the outer layer with conductivity 0.08 W/mK. The respective thickness of the inner, middle and outer layers are 25 cm, 5 cm and 3 cm respectively. The inside and outside wall temperatures are 600°C and 50°C respectively. Draw the equivalent electrical circuit for conduction of heat through the wall and find thermal resistance heat flow/m^2 and interface temperatures.

Given:-



Thermal conductivity of inner layer, $K_1 = 8.5$ W/mK

Thermal conductivity of middle layer, $K_2 = 0.25$ W/mK

Thermal conductivity of outer layer, $K_3 = 0.08$ W/mK

Inner layer thickness, $L_1 = 25\,cm = 0.25\,m$

Middle layer thickness, $L_2 = 5\,cm = 0.05\,m$

Outer layer thickness, $L_3 = 3\,cm = 0.03\,m$

Inside wall temp., $T_1 = 600^\circ c + 273 = 873\,K$

outside wall temp., $T_4 = 50^\circ c + 273 = 323\,K$

To find:

(i) Equivalent electrical circuit

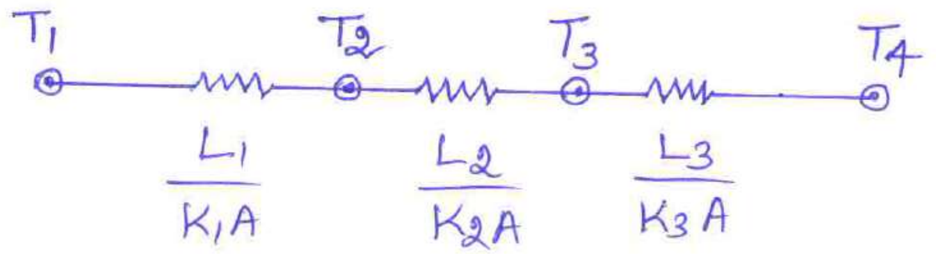
(ii) Heat flow per m^2

(iii) Thermal resistance

(iv) Interface temperatures

Solution:-

(i) Equivalent electrical circuit for conduction



(ii) Heat flow per m^2

From HMT DB page. 43 & 44

$$Q = \frac{\Delta T_{overall}}{R}$$

where $\Delta T = T_a - T_b$ = Inside temp. - outside temp.

$$\therefore \Delta T = T_1 - T_4$$

$$R = \frac{1}{h_a A} + \frac{L_1}{K_1 A} + \frac{L_2}{K_2 A} + \frac{L_3}{K_3 A} + \frac{1}{h_b A}$$

h_a & h_b values are not given. So, neglect

the first and last terms

$$\therefore\ R = \frac{L_1}{K_1 A} + \frac{L_2}{K_2 A} + \frac{L_3}{K_3 A}$$

$$Q = \frac{T_1 - T_4}{\frac{L_1}{K_1 A} + \frac{L_2}{K_2 A} + \frac{L_3}{K_3 A}}$$

$$\frac{Q}{A} = \frac{T_1 - T_4}{\frac{L_1}{K_1} + \frac{L_2}{K_2} + \frac{L_3}{K_3}}$$

$$\frac{Q}{A} = \frac{873 - 323}{\frac{0.25}{8.5} + \frac{0.05}{0.25} + \frac{0.03}{0.08}}$$

$$\frac{Q}{A} = 909.97\ W/m^2$$

(iii) Thermal resistance

$$R = \frac{L_1}{K_1 A} + \frac{L_2}{K_2 A} + \frac{L_3}{K_3 A}$$

For unit Area,

$$R = \frac{L_1}{K_1} + \frac{L_2}{K_2} + \frac{L_3}{K_3}$$

$$R = \frac{0.25}{8.5} + \frac{0.05}{0.25} + \frac{0.03}{0.08}$$

$$R = 0.604\ K/W$$

(iv) Interface temperatures

w.K.T.

$$Q = \frac{T_1 - T_4}{R} = \frac{T_1 - T_2}{R_1} = \frac{T_2 - T_3}{R_2} = \frac{T_3 - T_4}{R_3} \quad (1)$$

$$① \Rightarrow Q = \frac{T_1 - T_2}{R_1} = \frac{T_1 - T_2}{\frac{L_1}{K_1 A}} = \frac{T_1 - T_2}{\frac{1}{A}\left[\frac{L_1}{K_1}\right]}$$

$$\frac{Q}{A} = \frac{T_1 - T_2}{\frac{L_1}{K_1}}$$

$$909.97 = \frac{873 - T_2}{\left[\frac{0.25}{8.5}\right]}$$

$$T_2 = 846.23 \text{ K}$$

$$① \Rightarrow Q = \frac{T_2 - T_3}{R_2} = \frac{T_2 - T_3}{\frac{L_2}{K_2 A}} = \frac{T_2 - T_3}{\frac{1}{A}\left[\frac{L_2}{K_2}\right]}$$

$$\frac{Q}{A} = \frac{T_2 - T_3}{\frac{L_2}{K_2}}$$

$$909.97 = \frac{846.23 - T_3}{\left[\frac{0.05}{0.25}\right]}$$

$$T_3 = 664.23 \text{ K}$$

Results:-

(i) Heat flow/m^2, $Q/A = 909.97$ W/m^2

(ii) Thermal resistance, $R = 0.604$ K/W

(iii) Interface temperatures, $T_2 = 846.23$ K

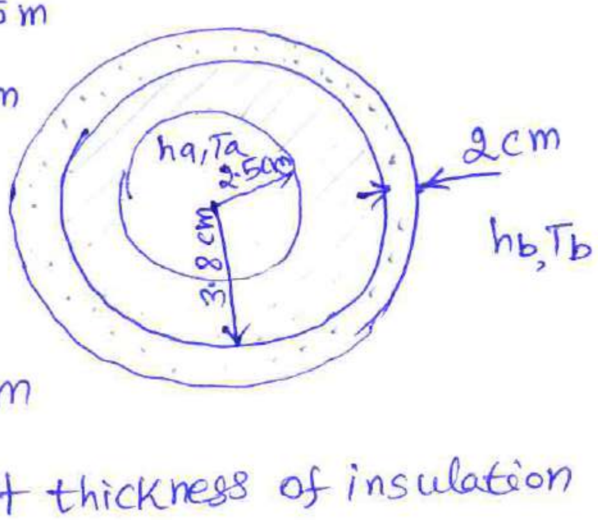
$T_3 = 664.23$ K

2. A steel tube of 5 cm I.D, 7.6 cm O.D. and $K = 15\ W/mK$ is covered with an insulation of thickness 2 cm and thermal conductivity $0.2\ W/mK$. A hot gas at $330°C$ and $h = 400\ W/m^2K$ flows inside the tube. The outer surface of the insulation is exposed to cold air at $30°C$ with $h = 60\ W/m^2K$. Assuming a tube length of 10 m, find the heat loss from the tube to the air. Also, find across which layer the largest temp. drop occurs.

Given:-

I.D. of steel tube, $d_1 = 5\,cm = 0.05\,m$

$\therefore$ Inner radius $= \frac{0.05}{2} = 0.025\,m$



O.D. of steel tube, $d_2 = 7.6\,cm = 0.076\,m$

$\therefore$ outer radius, $r_2 = \frac{0.076}{2} = 0.038\,m$

Insulation layer outer radius $\Big\}\ r_3 = r_2 +$ thickness of insulation

$r_3 = 0.038 + 0.02 = 0.058\,m$

Thermal conductivity of steel, $K_1 = 15\ W/mK$

Thermal conductivity of insulation, $K_2 = 0.2\ W/mK$

Hot gas temperature (inside), $T_a = 330°C + 273 = 603\,K$

H.T. co-efficient at inner side, $h_a = 400\ W/m^2K$

Ambient air temp. (outside), $T_b = 30°C + 273 = 303\,K$

H.T. co-efficient at outer side, $h_b = 60\ W/m^2K$

Length of the tube, $L = 10\,m$

To find:-
(i) Heat loss (Q), (ii) Largest temp. drop layer

Solution:- From HMTDB, pages 43 & 45

Heat flow, $Q = \frac{\Delta T_{overall}}{R}$, where $\Delta T = T_a - T_b$

$$R = \frac{1}{2\pi L}\left[\frac{1}{h_a r_1} + \frac{1}{K_1}\ln\left[\frac{r_2}{r_1}\right] + \frac{1}{K_2}\ln\left[\frac{r_3}{r_2}\right] + \frac{1}{h_b r_3}\right]$$

$$Q = \frac{T_a - T_b}{\frac{1}{2\pi L}\left[\frac{1}{h_a r_1} + \frac{1}{K_1}\ln\left[\frac{r_2}{r_1}\right] + \frac{1}{K_2}\ln\left[\frac{r_3}{r_2}\right] + \frac{1}{h_b r_3}\right]}$$

$$Q = \frac{603 - 303}{\frac{1}{2\pi \times 10}\left[\frac{1}{400 \times 0.025} + \frac{1}{15}\ln\left[\frac{0.038}{0.025}\right] + \frac{1}{0.2}\ln\left[\frac{0.058}{0.038}\right] + \frac{1}{60 \times 0.058}\right]}$$

$$Q = 7451.72\ W$$

W.K.T, Interface temperatures,

$$Q = \frac{T_a - T_b}{R} = \frac{T_a - T_1}{R_a} = \frac{T_1 - T_2}{R_1} = \frac{T_2 - T_3}{R_2} = \frac{T_3 - T_b}{R_3}$$

$$Q = \frac{T_a - T_1}{R_a} = \frac{T_a - T_1}{\frac{1}{2\pi L}\left[\frac{1}{h_a r_1}\right]}$$

$$7451.72 = \frac{T_a - T_1}{\frac{1}{2\pi \times 10} \times \left[\frac{1}{400 \times 0.025}\right]}$$

$$T_a - T_1 = 118.59\ K$$

Temp. drop across hot gas, $T_a - T_1 = 118.59\ K$

$$Q = \frac{T_1 - T_2}{R_1} = \frac{T_1 - T_2}{\frac{1}{2\pi L}\left[\frac{1}{K_1}\ln\left(\frac{r_2}{r_1}\right)\right]}$$

$$7451.72 = \frac{T_1 - T_2}{\frac{1}{2\pi \times 10}\left[\frac{1}{15}\ln\left[\frac{0.038}{0.025}\right]\right]}$$

$$T_1 - T_2 = 3.31\ K$$

Temp. drop across the steel tube, $T_1 - T_2 = 3.31K$

$$Q = \frac{T_2 - T_3}{R_2} = \frac{T_2 - T_3}{\frac{1}{2\pi L}\left[\frac{1}{K_2}\ln\left(\frac{r_3}{r_2}\right)\right]}$$

$$7451.72 = \frac{T_2 - T_3}{\frac{1}{2\pi \times 10}\left[\frac{1}{0.2}\ln\left(\frac{0.058}{0.038}\right)\right]}$$

$$T_2 - T_3 = 250.75\ K$$

Temp. drop across the insulation, $T_2 - T_3 = 250.75\ K$

$$Q = \frac{T_3 - T_b}{R_3} = \frac{T_3 - T_b}{\frac{1}{2\pi L}\left[\frac{1}{h_b r_3}\right]}$$

$$7451.72 = \frac{T_3 - T_b}{\frac{1}{2\pi \times 10}\left[\frac{1}{60 \times 0.058}\right]}$$

$$T_3 - T_b = 34.07\ K$$

Temp. drop across the outside air, $T_3 - T_b = 34.07K$

Result:

(i) Heat loss, $Q = 7451.72\ W$

(ii) Insulation layer is occurs largest temperature drop.

$$T_2 - T_3 = 250.75\ K$$

3. A hollow sphere 10 cm I.D. and 30 cm O.D. of ... a material having thermal conductivity 50 W/mK is used as a container for a liquid chemical mixture. Its inner and outer surface temperatures are 300°c and 100°c respectively. Determine the heat flow rate through the sphere. Also estimate the temperature at a point quarter of the way b/w the inner and outer surfaces.

Given:-

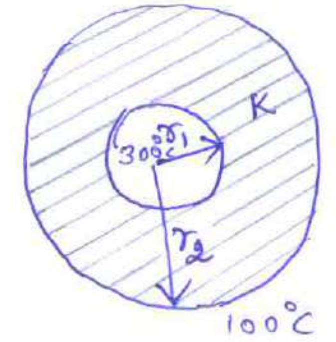
$T_1 = 300°C + 273 = 573\ K$

$T_2 = 100°C + 273 = 373\ K$

$K = 50\ W/mK$

$r_1 = \frac{10}{2} = 5\ cm = 0.05\ m$

$r_2 = \frac{30}{2} = 15\ cm = 0.15\ m$



To find:- Heat flow rate & Interior temp. at quarter-way

Solution:- From HMT DB P. 43 & 45

$$Q = \frac{\Delta T_{overall}}{R}, \quad \text{Here } \Delta T = T_1 - T_2$$

Thermal resistance of sphere

$$R = \frac{1}{4\pi}\left\{\frac{1}{h_a r_1^2} + \frac{1}{K_1}\left[\frac{1}{r_1} - \frac{1}{r_2}\right] + \frac{1}{h_b r_2^2}\right\}$$

The values of h_a & h_b are not given. So, neglect the first and last terms.

$$\therefore R = \frac{1}{4\pi}\left\{\frac{1}{K_1}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]\right\}$$

$$\therefore Q = \frac{4\pi(T_1 - T_2)}{\frac{1}{K_1}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]} = \frac{4\pi(573-373)}{\frac{1}{50}\left[\frac{1}{0.05} - \frac{1}{0.15}\right]} = 9420\ W$$

①

Radius of 1/4th way, r Thickness $= r_2 - r_1 = 15-5 = 10$ c

$$r = r_1 + \frac{1}{4} \times \text{thickness} = 5 + \left[\frac{1}{4} \times 10\right] = 7.5\,cm = 0.075\,m$$

Temp. at 1/4th way, T

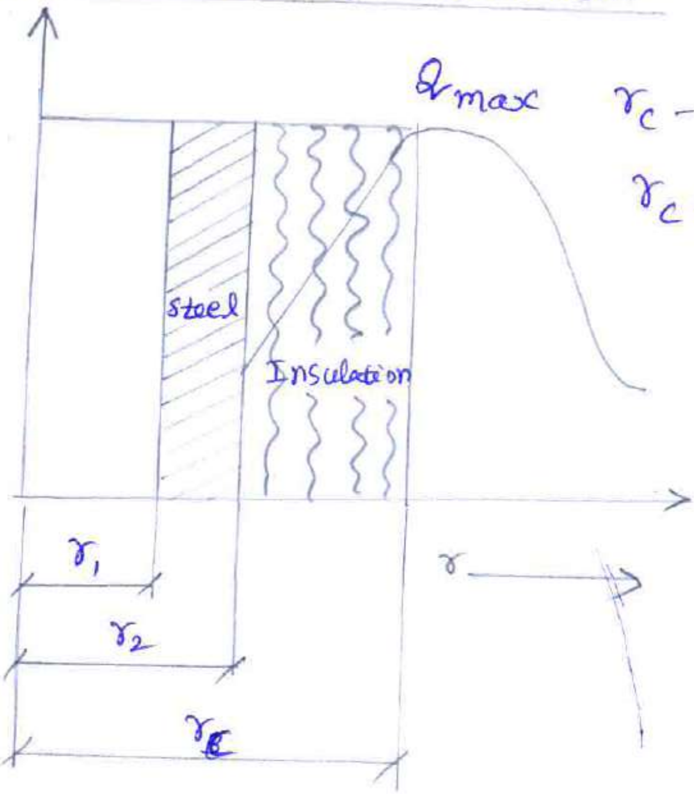
Replace $T_2 = T$ and $r_2 = r$ in eqn (1), we get

$$Q = \frac{4\pi(T_1 - T)}{\frac{1}{K_1}\left[\frac{1}{r_1} - \frac{1}{r}\right]}$$

$$9420 = \frac{4\pi(573 - T)}{\frac{1}{50}\left[\frac{1}{0.05} - \frac{1}{0.075}\right]}$$

$$T = 473\,K \text{ (or) } 200^\circ C$$

Critical radius of insulation:-



r_c - Critical radius

$r_c - r_2$ = Critical thickness

Addition of insulating material on a surface always reduces the amount of heat flow to the ambient.

At a certain thickness of insulation to the outside surface of cylindrical or spherical walls does not reduce the heat loss. That radius of insulation is called critical radius of insulation (r_c)

At critical radius (r_c) the heat transfer is maximum.

Internal heat generation - Formulae

For plane wall

1. Surface temp., $T_w = T_\infty + \frac{\dot{q}L}{2h}$
2. Max. temp., $T_{max} = T_w + \frac{\dot{q}L^2}{8K}$

T_∞ - Fluid temp.

$\dot{q}$ - Heat generation, W/m^3

L - Thickness, m

h - Heat transfer co-efficient, W/m^2K

K - Thermal conductivity, W/mK

For cylinder

1. $\dot{q} = \frac{Q}{V}$, where V = volume $= \pi r^2 \cdot L$
2. $T_{max} = T_w + \frac{\dot{q} r^2}{4K}$
3. Surface temp., $T_w = T_\infty + \frac{r\dot{q}}{2h}$

For sphere

Temp. at the centre, $T_c = T_w + \frac{\dot{q} r^2}{6K}$

4. A plane wall 10 cm thick generates heat at the rate of 4×10^4 W/m^3 when an electric current is passed through it. The convective heat transfer co-efficient b/w each face of the wall and the ambient air is 50 W/m^{2}K. Determine
(a) the surface temperature
(b) the max. temp. in the wall
Assume the ambient air temp. to be 20°C and the thermal conductivity of the wall material to be 15 W/mK.

Given:-

Thickness, $L = 10\text{ cm} = 0.1\text{ m}$

Heat generation, $\dot{q} = 4 \times 10^4$ W/m^3

H.T. co-efficient, $h = 50$ W/m^{2}K

Ambient air temp., $T_\infty = 20°C$

Thermal conductivity, $K = 15$ W/mK

To find

(a) surface temp., (b) Max. temp. in the wall

Solution:-

(a) surface temp., $T_w = T_\infty + \frac{\dot{q}L}{2h}$

$$T_w = 20°C + \frac{4 \times 10^4 \times 0.1}{2 \times 50}$$

$$T_w = 60°C \text{ or } 333\text{K}$$

(b) Max. temp., $T_{max} = T_w + \frac{\dot{q}L^2}{8K}$

$$= 60°C + \frac{4 \times 10^4 \times (0.1)^2}{8 \times 15}$$

$$= 63.33°C \text{ or } 336.33\text{K}$$

5. An electrical transmission line made of a 25 mm diameter annealed copper wire carries 200 A and has a resistance of $0.4 \times 10^{-4}\,\Omega/cm$ length. If the surface temp. is 200°C and the ambient air temp. is 10°C, determine the heat transfer co-efficient b/w the wire surface and the ambient air and the maximum temperature in the wire. Assume $K = 150\,W/mK$.

Given: $d = 25\,mm$; $r = \frac{25}{2} = 12.5\,mm = 12.5 \times 10^{-3}\,m$

$$I = 200\,A$$

$$R = 0.4 \times 10^{-4}\,\Omega/cm = \frac{0.4 \times 10^{-4}}{10^{-2}}\,\Omega/m$$

$$= 0.4 \times 10^{-2}\,\Omega/m$$

Surface temp., $T_w = 200°C$

Ambient temp., $T_\infty = 10°C$

$$K = 150\,W/mK$$

To find:- h, T_{max}

Solution:-

Heat loss in the electrical transmission line $\Big\}\ Q = I^2 R$

$$Q = (200)^2 \times 0.4 \times 10^{-2} = 160\,W/m$$

Heat generation, $\dot{q} = \frac{Q}{V}$ $\quad [\because V = \pi \cancel{4} r^2 L]$

$$\dot{q} = \frac{Q}{\pi r^2 L} = \frac{160}{\pi (0.0125)^2 \times 1}$$

$$\dot{q} = 325949.3\,W/m^3$$

$$T_{max} = T_w + \frac{\dot{q} r^2}{4K}$$

$$= 200°C + \frac{325949.3 \times (0.0125)^2}{4 \times 150}$$

$$T_{max} = 200.08\ °C$$

W.K.T., $T_w = T_\infty + \frac{r \dot{q}}{2h}$

$$200 = 10 + \frac{0.0125 \times 325949.3}{2 \times h}$$

$$200 - 10 = \frac{0.0125 \times 325949.3}{2 \times h}$$

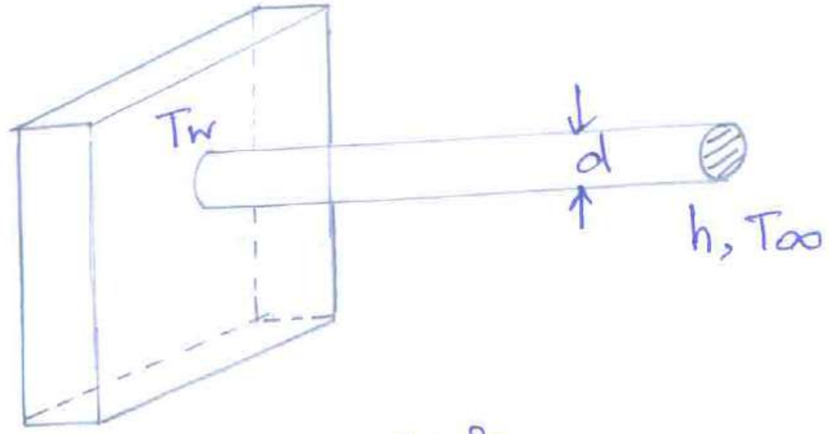
$$h = \frac{0.0125 \times 325949.3}{2 \times 190}$$

$$h = 10.72\ W/m^2K$$

Extended surfaces

6. One end of a very long aluminium rod is connected to a wall at 140°C, while the other end protrudes into a room whose air temp. is 15°C. The rod is 3 mm in diameter and the heat transfer coefficient b/w the rod surface and environment is 300 W/m²K. Estimate the total heat dissipated by the rod taking its thermal conductivity as 150 W/mK.

Given:-



Base temp., $T_w = 140°C$

Surrounding temp., $T_\infty = 15°C$

Fin diameter, $d = 3\ mm = 0.003\ m$

H.T. co-efficient, $h = 300\ W/m^2K$

Thermal conductivity, $K = 150\ W/mK$

Solution:- The rod is very long. (Long pin)

From HMT DB, P. 49

$$Q = [T_b - T_\infty]\sqrt{hPKA}$$

Area of Aluminium rod, $A = \frac{\pi}{4}d^2 = \frac{\pi}{4} \times (0.003)^2 = 7.06 \times 10^{-6}$ m

Perimeter of Al. rod, $P = \pi . d = \pi \times 0.003 = 9.424 \times 10$

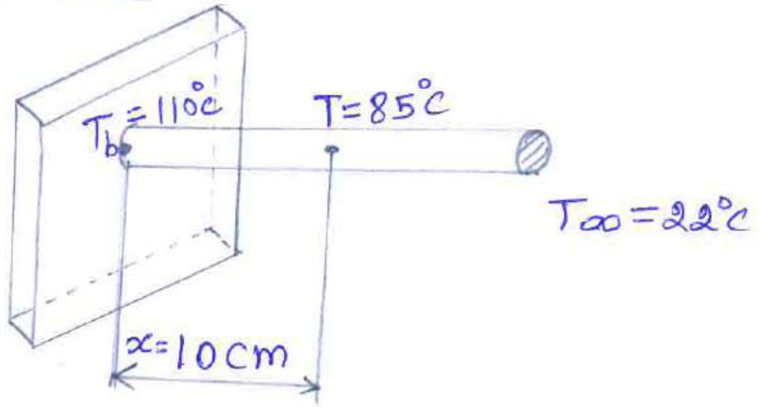
$$Q = (140-15)\sqrt{300 \times 9.424 \times 10^{-3} \times 150 \times 7.06 \times 10^{-6}}$$

$$Q = 6.843\ W$$

7. In an experiment to determine the thermal Conductivity of a long solid 2.5 cm diameter rod its base is placed in a furnace with a large portion of it projecting into the room air at 22°C. After steady state conditions prevail,

the temperatures at two points, 10 cm apart ar found to be 110°c and 85°c respectively. The Convective heat transfer co-efficient b/w the rod surface and the surrounding air is 28.4 W/m²K. Determine the thermal conductivity of the rod material.

Given:



Diameter of the solid rod, $d = 2.5cm = 0.025m$

Room temp., $T_\infty = 22°c$

Distance b/w two points, $x = 10 cm = 0.1 m$

Temp. at point 1 (furnace wall) $\} = 110°c$
T_b

Temp. at point 2, $T = 85°c$

H.T. co-efficient, $h = 28.4 W/m^2K$

To find:- Thermal conductivity, K

Solution:- Rod length is not given.
Since the rod is very long.

From HMT DB, P. 49

$$\frac{T - T_\infty}{T_b - T_\infty} = e^{-mx}$$

$$\frac{85 - 22}{110 - 22} = e^{-(0.1m)}$$

$$0.7159 = e^{-(0.1m)}$$

$$\ln(0.7159) = -0.1 \times m$$

$$-0.3342 = -0.1 \times m$$

$$m = 3.342$$

$$m = \sqrt{\frac{hP}{KA}}$$

Area of solid rod, $A = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times (0.025)^2$

$$A = 4.91 \times 10^{-4} \, m^2$$

Perimeter of solid rod, $P = \pi \cdot D = \pi \times 0.025$

$$P = 7.85 \times 10^{-2} \, m$$

$$\therefore m = \sqrt{\frac{28.4 \times 7.85 \times 10^{-2}}{K \times 4.91 \times 10^{-4}}}$$

Take square on both sides,

$$m^2 = \frac{28.4 \times 7.85 \times 10^{-2}}{K \times 4.91 \times 10^{-4}}$$

$$K = \frac{28.4 \times 7.85 \times 10^{-2}}{m^2 \times 4.91 \times 10^{-4}}$$

$$K = \frac{28.4 \times 7.85 \times 10^{-2}}{(3.342)^2 \times 4.91 \times 10^{-4}}$$

$$K = 406.53 \, W/mK$$

8. An aluminium rod 2.5 cm in diameter and 16 cm long protrudes from a wall which is maintained at 260°C. The rod is exposed to an environment at 16°C. The convective heat transfer co-efficient is 15 W/m²K. Calculate the heat lost by the rod. Take K = 190 W/mK.

Given:- Length of the rod is given. Therefore short fi

Diameter of the rod, $d = 2.5\text{ cm} = 0.025\text{ m}$

Length of the rod, $L = 16\text{ cm} = 0.16\text{ m}$

Base temperature (wall temp.), $T_b = 260°C + 273 = 533\text{ K}$

Surrounding temp. (environment temp.), $T_\infty = 16°C + 273 = 289\text{ K}$

Heat transfer co-efficient, $h = 15\text{ W/m}^2\text{K}$

Thermal conductivity, $K = 190\text{ W/mK}$

To find:- Heat lost by the rod, Q

Solution:-

$$\frac{L}{d} = \frac{0.16}{0.025} = 6.4 < 30$$

This is short fin. Assume the end is insulated

From HMT DB P49

Heat transferred [short fin, end insulated]

$$Q = (hPKA)^{1/2}(T_b - T_\infty)\tanh(mL)$$

Area of the aluminium rod, $A = \frac{\pi}{4}d^2$

$$A = \frac{\pi}{4}\times(0.025)^2 = 4.9\times10^{-4}\text{ m}^2$$

Perimeter of the aluminium rod, $P = \pi \cdot D$

$$P = \pi \times 0.025 = 0.0785\text{ m}$$

$$m = \sqrt{\frac{hP}{KA}} = \sqrt{\frac{15 \times 0.0785}{190 \times 4.9 \times 10^{-4}}} = 3.5$$

$$Q = \left(15 \times 0.0785 \times 190 \times 4.9 \times 10^{-4}\right)^{1/2} \times (533-289) \times \tanh(3.5 \times 0.16)$$

$Q = 41$ Watts

Fin efficiency:- The efficiency of a fin is defined as the ratio of the actual heat transferred by fin to the maximum possible heat transferable by the fin.

$$\eta_{fin} = \frac{Q_{fin}}{Q_{max}}$$

Fin effectiveness:- It is defined as the ratio of heat transfer with fin to heat transfer without fin.

$$\text{Effectiveness}_{fin} = \frac{Q_{\text{with fin}}}{Q_{\text{without fin}}}$$

Unsteady Heat conduction - Lumped analysis

9. A copper plate 2 mm thick is heated upto 400°c and quenched into water at 30°c. Find the time required for the plate to reach the temp. of 50°c. Heat transfer co-efficient is 100 W/m^2K. Density of copper is 8800 Kg/m^3. Specific heat of copper = 0.36 KJ/KgK. Plate dimensions 30×30

Given:-

Plate thickness, $L = 2\text{ mm} = 0.002\text{ m}$

Initial temp., $T_0 = 400°c + 273 = 673\text{ K}$

Final temp. $T_\infty = 30°c + 273 = 303\text{ K}$

Interior temp, $T = 50°C$

H.T. co-efficient, $h = 100\ W/m^2K$

Density of copper, $\rho = 8800\ Kg/m^3$

Specific heat, $C_p = 360\ J/KgK$

Plate dimensions $= 30 \times 30\ cm = 0.3 \times 0.3\ m$

To find:- Time required to reach 50°C

Solution:- From HMT DB, P2

Thermal conductivity of copper, $K_{copper} = 386\ W/mK$

For slab

Characteristic Length, $L_c = \frac{L}{2} = \frac{0.002}{2} = 0.001\ m$

W.K.T., Biot number, $B_i = \frac{hL_c}{K} = \frac{100 \times 0.001}{386}$

$$Bi = 2.59 \times 10^{-4} < 0.1$$

Bi value is less than 0.1. So, this is lumped analysis problem.

For lumped parameter system, From HMT DB P.57

$$\frac{T - T_\infty}{T_0 - T_\infty} = e^{\left[\frac{-hA}{C_P \times V \times \rho} \times t\right]}$$

W.K.T. $L_c = \frac{V}{A}$

$$\frac{T - T_\infty}{T_0 - T_\infty} = e^{\left[\frac{-h}{C_P \times L_c \times \rho} \times t\right]}$$

$$\frac{323 - 303}{673 - 303} = e^{\left[\frac{-100}{360 \times 0.001 \times 8800} \times t\right]}$$

$$\ln(0.0545) = \frac{-100}{360 \times 0.001 \times 8800} \times t$$

$$t = 92.43\ sec.$$

Lumped heat analysis: In a newtonian heating or cooling process the temp. is considered to be uniform at a given time.

Biot number (B_i) The ratio of internal conduction resistance to the surface convection resistance is known as Biot number. $B_i = \frac{hL_c}{K}$, where $L_c = \frac{V}{A}$

Characteristic Length (L_c) $L_c = \frac{V}{A}$

For slab, $L_c = \frac{L}{2}$, where L - Thickness of slab

For cylinder, $L_c = \frac{R}{2}$, where R - Radius of cylinder

For sphere, $L_c = \frac{R}{3}$, where R - Radius of sphere

For cube, $L_c = \frac{L}{6}$, where L - thickness of cube

10 A steel ball ($C_p = 0.46$ KJ/KgK and $K = 35$ W/mK) having 5cm diameter and initially at a uniform temp. of 450°C is suddenly placed in a control environment in which the temp is maintained at 100°C. Calculate the time required for the ball to attained a temperature of 150°C. Take $h = 10$ W/m²K

Given:-

Specific heat, $C_p = 0.46 \times 10^3$ J/KgK

Thermal conductivity, $K = 35$ W/mK

Diameter of the ball, $D = 5$ cm $= 0.05$ m

Radius of the ball, $R = \frac{D}{2} = \frac{0.05}{2} = 0.025$ m

Initial temp., $T_0 = 450°C + 273 = 723$ K

Final temp., $T_\infty = 100°C + 273 = 373$ K

Interior temp., $T = 150°C + 273 = 423$ K

H.T. co-efficient, $h = 10$ W/m²K

To find :- Time required to reach 150°C

Solution:- From HMT DB, P1

Density of steel, $\rho_{steel} = 7833$ Kg/m³

For sphere, characteristic length, $L_c = \frac{R}{3} = \frac{0.025}{3}$

W.K.T., Biot Number, $B_i = \frac{hL_c}{K}$ $\quad L_c = 8.33 \times 10^{-3}$ m

$$B_i = \frac{10 \times 8.33 \times 10^{-3}}{35} = 2.38 \times 10^{-3} < 0.1$$

Bi value is less than 0.1, so, this is lumped analysis problem.

For lumped parameter system, From HMTDB P57

$$\frac{T - T_\infty}{T_0 - T_\infty} = e^{\left[\frac{-hA}{C_p \times V \times \rho} \times t\right]} \quad \text{W.K.T. } L_c = \frac{V}{A}$$

$$\therefore \frac{T - T_\infty}{T_0 - T_\infty} = e^{\left[\frac{-h}{C_p \times L_c \times \rho} \times t\right]}$$

$$\frac{423 - 373}{723 - 373} = e^{\left[\frac{-10}{0.46 \times 10^3 \times 8.33 \times 10^{-3} \times 7833} \times t\right]}$$

$$\ln(0.1428) = \frac{-10}{0.46 \times 10^3 \times 8.33 \times 10^{-3} \times 7833} \times t$$

$$-1.9463 = -0.000333\, t$$

$$t = \frac{-1.9463}{-0.000333} = 5844.74 \text{ sec.}$$

Semi Infinite and Infinite Solids - Use of Heisler's Charts

11. A slab of aluminium 10 cm thick is originally at a temperature of 500 °C. It is suddenly immersed in a liquid at 100°C resulting in heat transfer coefficient of 1200 W/m²K. Determine the temp. at the centerline and the surface 1 minute after the immersion. Also calculate the total thermal energy removed per unit area of the slab during this period. The properties of aluminium for the given conditions are: $\alpha = 8.4 \times 10^{-5}$ m²/sec, $K = 215$ W/mK, $\rho = 2700$ Kg/m³, $C_p = 0.9$ KJ/KgK.

Given data:

Thickness, $L = 10\ cm = 0.1\ m$

Initial temp., $T_i = 500 + 273 = 773\ K$

Liquid temp., $T_\infty = 100 + 273 = 373\ K$

H.T. coefficient, $h = 1200\ W/m^2K$

Thermal diffusivity, $\alpha = 8.4 \times 10^{-5}\ m^2/s$

Thermal conductivity, $K = 215\ W/mK$

Density, $\rho = 2700\ Kg/m^3$

Specific heat, $C_p = 0.9 \times 10^3\ J/KgK$

time, $t = 1\ min = 60$ seconds

To find:

* Temperature at the centerline after 1 minute
* Temperature at the surface
* Total heat energy removed

Solution:-

(i) Temp. at the centerline (T_0)

$Bi = \frac{hL_c}{K}$ $\qquad L_c = \frac{L}{2}$ (for slab)

$Bi = \frac{1200 \times 0.05}{215}$ $\qquad L_c = \frac{0.1}{2} = 0.05\,m$

$Bi = 0.279$

Biot Number Value is in between 0.1 & 100

$0.1 < Bi < 100$

∴ This is infinite Solid type Problem.
So, Heisler chart to be used to solve this Problem.

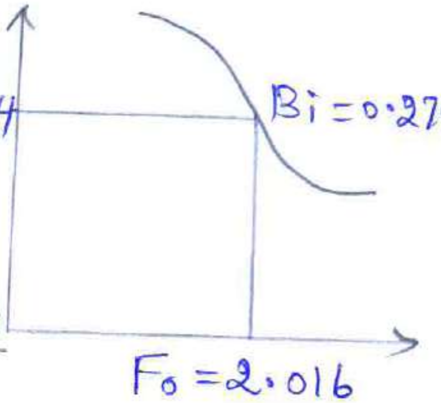
From HMT DB, P65

Fourier Number $= \frac{\alpha t}{L_c^2} = \frac{8.4 \times 10^{-5} \times 60}{(0.05)^2}$

$F_o = 2.016$ [X axis find this value]

$Bi = \frac{hL_c}{K} = 0.279$

[on curve find this value] $\frac{T_o - T_\infty}{T_i - T_\infty} = 0.64$



From HMT DB, P65

Heisler chart, for $F_o = 2.01$ & $Bi = 0.279$

$\frac{T_o - T_\infty}{T_i - T_\infty} = 0.64$

$\frac{T_o - 373}{773 - 373} = 0.64$

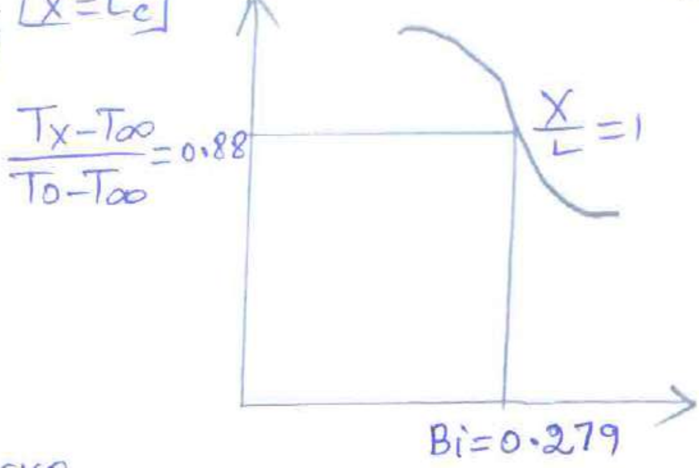
$T_o - 373 = 0.64 \times (773 - 373)$

$T_o = 256 + 373 = 629\,K$

(ii) Surface temperature $[x = L_c]$ (T_x)

From HMTDB, P66,

Heisler Chart



$B_i = 0.279 \rightarrow X$ axis

$\frac{X}{L_c} = \frac{L_c}{L_c} = \frac{1}{1} = 1$

$\therefore \frac{X}{L} = 1 \rightarrow$ on curve

$$\frac{T_x - T_\infty}{T_0 - T_\infty} = 0.88$$

$$\frac{T_x - 373}{629 - 373} = 0.88$$

$$T_x - 373 = 0.88(629 - 373)$$

$$T_x = 225.28 + 373 = 598.28 \text{ K}$$

Surface temp., $T_x = 598.28$ K

(iii) Total heat energy removed (Q)

From Heisler chart, HMTDB P67

$$F_0 = \frac{h^2 \alpha t}{K^2} = \frac{(1200)^2 \times 8.4 \times 10^{-5} \times 60}{(215)^2}$$

$F_0 = 0.157 \rightarrow$ X-axis

$Bi = 0.279 \rightarrow$ on curve

$$\frac{Q}{Q_0} = 0.34$$

From HMT DB P63

$$Q_0 = \rho C_p L (T_i - T_\infty)$$

$$Q_0 = 2700 \times 0.9 \times 10^3 \times 0.1 (773 - 373) = 97.2 \times 10^6 \text{ J/m}^2$$

$$\frac{Q}{Q_0} = 0.34$$

$$\therefore Q = 0.34 \times Q_0$$

$$Q = 0.34 \times 97.2 \times 10^6$$

$$Q = 33.04 \times 10^6 \, J/m^2$$

Total heat energy removed per unit area $\Big\}$ $Q = 33.04 \times 10^6 \, J/m^2$

UNIT-II CONVECTION

Convection:- convection is a process of heat transfer that will occur between a solid surface and a fluid medium when they are at different temperatures.

Newton's law of convection:-

Heat transfer from the moving fluid to solid surface is given by the equation,

$Q = hA(T_W - T_\infty)$

where, h - Local heat transfer co-efficient in W/m^2K

A - Surface area in m^2,

T_W - Surface (or) wall temp. in K,

T_∞ - Temp. of the fluid in K.

Types of convection:-

* Free convection * Forced convection

Free convection:-

If the fluid motion is produced due to change in density resulting from temperature gradients, the mode of heat transfer is said to be free or natural convection.

Forced convection:-

If the fluid motion is artificially created by means of an external force like a blower or fan, that type of heat transfer is known as forced convection.

Boundary layer concept:

The concept of a boundary layer as proposed by Prandtl, forms the starting point for the simplification of the equations of motion and energy.

When a real fluid, flows along a stationary solid boundary, a layer of fluid which comes in contact with the boundary surface. The layer adjacent to the boundary is known as boundary layer. It is formed whenever there is relative motion between the boundary and the fluid. In this concept, the flow field over a body is divided into two regions.

1. A thin region near the body called the boundary layer, where the velocity and temp. gradients are large.
2. The region outside the boundary layer where velocity and temp. gradients are very nearly equal to their free stream values.

Types of boundary layer

1. Hydro dynamic boundary layer
   (or)
   Velocity boundary layer
2. Thermal boundary layer

Hydrodynamic boundary layer:

Velocity of the fluid is less than 99% of free stream velocity.

Thermal boundary layer:

Temperature of the fluid is less than 99% of free stream temperature.

Dimensional analysis:-

In dimensional analysis, the various physical quantities used in fluid phenomenon can be expressed in terms of fundamental quantities. These are mass (M), length (L), time (T) and temperature (θ)

For Example,

$$\text{Velocity, } V = \frac{\text{Distance}}{\text{Time}} = \frac{L}{T} = LT^{-1}$$

Length, L = Distance = L

Area, A = Distance $\times$ Distance = L^2

Buckingham π Theorem: If there are n variables in a dimensionally homogeneous equation and if these contain m fundamental dimensions, then the variables are arranged into (n-m) dimensionless terms. These dimensionless terms are called π terms.

$$Re = \frac{\rho U^{\cancel{2}} L^{\cancel{2}}}{\mu \cancel{U} \cancel{L}} = \frac{\rho U L}{\mu} = \frac{UL}{\nu} \qquad \left[\because \nu = \frac{\mu}{\rho}\right]$$

$$Pr = \frac{\mu C_p}{K} = \frac{\nu}{\alpha}$$

$$Nu = \frac{q_{conv.}}{q_{cond.}} = \frac{hA\Delta T}{KA\frac{\Delta T}{L}} = \frac{h}{K/L} = \frac{hL}{K}$$

Forced convection - Procedure to solve the problem

1. If velocity is given, that type of problem is consider as forced convection problem.
2. Film temp., $T_f = \frac{T_w + T_\infty}{2}$
3. If 'Re' value is less than 5×10^5 (Flat plate), then the flow is laminar

$$Re = \frac{UL}{\nu} < 5 \times 10^5 \rightarrow \text{Laminar flow (Flat plate)}$$

$$Re = \frac{UL}{\nu} > 5 \times 10^5 \rightarrow \text{Turbulent flow (Flat plate)}$$

For Flat plate laminar flow

From HMTDB, P112 (6th edition)

1. Local Nusselt Number

$$Nu_x = 0.332 (Re)^{0.5} (Pr)^{0.333}$$

$$Nu_x = \frac{h_x \cdot L}{K}$$

2. Average heat transfer co-efficient

$$h = 2 \times h_x$$

3. Heat transfer, $Q = hA(T_w - T_\infty)$

4. Hydrodynamic boundary layer thickness

$$\delta_{hx} = 5 \times x \times (Re)^{-0.5}$$

5. Thermal boundary layer thickness

$$\delta_{Tx} = \delta_{hx} (Pr)^{-0.333}$$

6. Local friction co-efficient

$$C_{fx} = 0.664 (Re)^{-0.5}$$

7. Average friction co-efficient

$$\bar{C}_{fL} = 1.328 (Re)^{-0.5}$$

Problem:-

1. Air at 20°C at atmospheric pressure flows over a flat plate at a velocity of 3 m/s. If the plate is 1 m wide and 80°C, calculate the following at $x = 300$ mm.

Given:-

Fluid temperature, $T_\infty = 20°C$

Velocity, $U = 3$ m/s

Wide, $w = 1$ m

Surface temperature, $T_w = 80°C$

Distance, $x = 300$ mm $= 0.3$ m

$x = L = 0.3$ m

To find:-

(i) Hydro dynamic boundary layer thickness

(ii) Thermal boundary layer thickness

(iii) Local friction co-efficient

(iv) Average friction co-efficient

(v) Local heat transfer co-efficient

(vi) Average heat transfer co-efficient

(vii) Heat transfer

Solution:-

W.K.T. Film temp., $T_f = \frac{T_w + T_\infty}{2} = \frac{80+20}{2} = 50°C$

From HMTDB P33, properties of air at 50°C

Density, $\rho = 1.093\ Kg/m^3$

Kinematic viscosity, $\nu = 17.95 \times 10^{-6}\ m^2/s$

Prandtl Number, $P_r = 0.698$

Thermal conductivity, $K = 0.02826\ W/mK$

W.K.T., $Re = \frac{UL}{\nu}$

$$Re = \frac{3 \times 0.3}{17.95 \times 10^{-6}}$$

$$Re = 5.01 \times 10^4 < 5 \times 10^5$$

$\therefore$ Flow is laminar.

For Flat plate laminar flow,

From HMTDB, P112

(i) Hydrodynamic boundary layer thickness

$$\delta_{hx} = 5 \times x \times (Re)^{-0.5}$$
$$= 5 \times 0.3 \times (5.01 \times 10^4)^{-0.5}$$
$$= 6.7 \times 10^{-3}\ m$$

(ii) Thermal boundary layer thickness

$$\delta_{Tx} = \delta_{hx} (Pr)^{-0.333}$$
$$= 6.7 \times 10^{-3} \times (0.698)^{-0.333}$$
$$= 7.5 \times 10^{-3}\ m$$

(iii) Local friction co-efficient

$$C_{fx} = 0.664 (Re)^{-0.5}$$

$$= 0.664 \times (5.01 \times 10^4)^{-0.5}$$

$$C_{fx} = 2.96 \times 10^{-3}$$

(iv) Average friction co-efficient

$$\bar{C}_{fL} = 1.328 (Re)^{-0.5}$$

$$= 1.328 (5.01 \times 10^4)^{-0.5}$$

$$= 5.9 \times 10^{-3}$$

(v) Local heat transfer co-efficient (h_x)

Local Nusselt Number

$$Nu_x = 0.332 (Re)^{0.5} (Pr)^{0.333}$$

$$= 0.332 (5.01 \times 10^4)^{0.5} (0.698)^{0.333}$$

$$= 65.9$$

W.K.T., $Nu_x = \frac{h_x L}{K}$

$$65.9 = \frac{h_x \times 0.3}{0.02826}$$

$$h_x = 6.20 \ W/m^2K$$

(vi) Average heat transfer co-efficient

$$h = 2 h_x = 2 \times 6.20$$

$$h = 12.4 \ W/m^2K$$

(vii) Heat transfer

$$Q = hA (T_w - T_\infty)$$

$$= 12.41 \times (1 \times 0.3)(80 - 20)$$

$$Q = 223.38 \text{ watts}$$

For Flat Plate, Turbulent flow

From HMTDB, P 113

1. Local Nusselt Number

$Nux = 0.0296 \, (Re)^{0.8} (Pr)^{0.333}$

$Nux = \frac{h_x \cdot L}{K}$

2. Average heat transfer co-efficient

$h = 1.25 \, h_x$

3. Boundary layer thickness

$\delta = 0.37 \times x \times (Re)^{-0.2}$

4. Local Skin friction co-efficient

$C_{fx} = 0.0576 \, (Re)^{-0.2}$

5. Heat transfer, $Q = hA(T_w - T_\infty)$

2. Air at 25°C flows over 1m x 3m (3m long) horizontal plate maintained at 200°c at 10 m/s. calculate the average heat transfer co-efficients for both laminar and turbulent regions. Take Re(critical) = 3.5×10^5

Given:

Fluid temp., $T_\infty = 25°C$

Length, $L = 3\,m$

Plate temp., $T_w = 200°c$

Velocity, $U = 10\,m/s$

Re (critical) $= 3.5 \times 10^5$

To find:

(i) Average H.T. co-efficient for laminar flow

(ii) Average H.T. co-efficient for turbulent flow

Solution:-

Film temperature, $T_f = \frac{T_w + T_\infty}{2} = \frac{200+25}{2} = 112.5°C$

From HMTDB P33, properties of air at 112.5°C

$\rho = 0.922\ Kg/m^3$; $\nu = 24.29 \times 10^{-6}\ m^2/s$; $Pr = 0.687$

$K = 0.03274\ W/mK$

For this problem, $Re(critical) = 3.5 \times 10^5$

$\therefore$ The flow is laminar upto $Re = 3.5 \times 10^5$

The flow is turbulent $Re > 3.5 \times 10^5$

(i) For laminar flow, From HMTDB, P112

Local Nusselt Number, $Nu_x = 0.332 (Re)^{0.5} (Pr)^{0.333}$

$Nu_x = 0.332 \times (3.5 \times 10^5)^{0.5} \times (0.687)^{0.333} = 173.33$

$Nu_x = \frac{h_x L}{K}$

$h_x = \frac{173.33 \times 0.03274}{3} = 1.89\ W/m^2K$

Local H.T. co-efficient, $h_x = 1.89\ W/m^2K$

Average H.T. co-efficient, $h = 2 \times h_x = 2 \times 1.89$

$h = 3.78\ W/m^2K$

(ii) For turbulent flow

$Re = \frac{UL}{\nu} = \frac{10 \times 3}{24.29 \times 10^{-6}} = 1.23 \times 10^6 > 3.5 \times 10^5$

$\therefore$ The flow is turbulent.

From HMTDB, P113

Local Nusselt Number, $Nu_x = 0.0296 (Re)^{0.8} (Pr)^{0.333}$

$Nu_x = 0.0296 \times [1.23 \times 10^6]^{0.8} \times (0.687)^{0.333}$

$Nu_x = 1945$

$$Nu_x = \frac{h_x \cdot L}{K}$$

$$h_x = \frac{1945 \times 0.03274}{3} = 21.22 \text{ W/m}^2\text{K}$$

Local H.T. Co-efficient $= 21.22 \text{ W/m}^2\text{K}$

Average H.T. Co-efficient for turbulent flow $\Big\}$ $h = 1.25\, h_x = 1.25 \times 21.22$

$$h = 26.525 \text{ W/m}^2\text{K}$$

For Flat Plate (Laminar, Turbulent combined - constant Wall temperature)

1. Average Nusselt Number, $Nu = Pr^{0.333}\left[0.037(Re)^{0.8} - 871\right]$

$$\text{Also, } Nu = \frac{hL}{K}$$

2. Average friction coefficient, $C_{fL} = 0.074(Re)^{-0.2} - 1742(Re)^{-1.0}$

3. Atmospheric air at 2°C and a free stream velocity of 20 m/s flows over a flat plate 1.5 m long that is maintained at a uniform temperature of 52°C. Calculate the average H.T. coefficient over the region where the boundary layer is laminar, the average H.T. coefficient over the entire length of the plate and the total heat transfer rate from the plate to the air over the length 1.5 m and width 1 m. Assume transition occurs at $Re(\text{critical}) = 2 \times 10^5$

Given: Fluid temp., $T_\infty = 2°C$

Velocity, $U = 20$ m/s

Length, $L = 1.5$ m

Plate surface temp., $T_W = 52°C$

Width, $W = 1$ m

Critical Reynold's Number, $Re_c = 2 \times 10^5$

To find.

(i) Average H.T. coefficient for laminar flow

(ii) Average H.T. coefficient for combined flow (laminar & turbulent flow); (iii) Heat transfer rate, Q

Solution:-

Film temperature, $T_f = \frac{T_w + T_\infty}{2} = \frac{52+2}{2} = 27°C$

From HMTDB, P33, properties of air at $27°C \simeq 25°C$

$\rho = 1.185\ kg/m^3$; $\nu = 15.53 \times 10^{-6}\ m^2/s$; $Pr = 0.702$

$K = 0.02634\ W/mK$

Here, $Re_c = 2 \times 10^5$

$\therefore$ Flow is laminar upto 2×10^5

$$Re = \frac{UL}{\nu} \Rightarrow 2\times10^5 = \frac{20 \times L}{15.53\times10^{-6}} \Rightarrow L = 0.155\ m$$

From HMTDB P112, For Flat Plate, laminar flow

Local Nusselt Number, $Nu_x = 0.332 (Re)^{0.5} (Pr)^{0.333}$

$Nu_x = 0.332 \times (2\times10^5)^{0.5} (0.702)^{0.333}$

$Nu_x = 131.97$

Also, $Nu_x = \frac{h_x \cdot L}{K}$

$$h_x = \frac{131.97 \times 0.02634}{0.155} = 22.42\ W/m^2K$$

Local H.T. Co-efficient, $h_x = 22.42\ W/m^2K$

Average H.T. Coefficient, $h = 2 \times h_x$

$h = 2 \times 22.42$

$h = 44.84\ W/m^2K$

For Flat Plate, laminar - turbulent combined flow

Reynold's Number for entire length, $Re_L = \frac{UL}{\nu}$

$$Re_L = \frac{20 \times 1.5}{15.53 \times 10^{-6}} = 1.93 \times 10^6 > 5 \times 10^5$$

$\therefore$ The flow is turbulent

Average Nusselt Number, $Nu = (Pr)^{0.333}\left[0.037(Re_L)^{0.8} - 871\right]$

$$Nu = (0.702)^{0.333}\left[0.037\,(1.93 \times 10^6)^{0.8} - 871\right]$$

$$Nu = 2737.18$$

Also, $Nu = \frac{hL}{K} \Rightarrow h = \frac{Nu \times K}{L} = \frac{2737.18 \times 0.02634}{1.5}$

Average H.T. co-efficient for combined flow, $h = 48.06\ W/m^2K$

Total heat transfer rate, $Q = hA(T_w - T_\infty)$

$$= h \times w \times L (T_w - T_\infty)$$

$$= 48.06 \times 1 \times 1.5 \times (52 - 2)$$

$$= 3604.5 \text{ Watts}$$

Flow through a cylinder - Internal flow

1. Bulk mean temperature, $T_m = \frac{T_{mi} + T_{mo}}{2}$

   T_{mi} - Inlet temp. in °C

   T_{mo} - outlet temp. in °C

2. Reynold's Number, $Re = \frac{UD}{\nu}$

   If $Re < 2300$, flow is laminar

   If $Re > 2300$, flow is turbulent

3. Laminar flow,

   Nusselt Number, $Nu = 3.66$

4. Turbulent flow

Nusselt Number, $Nu = 0.023 (Re)^{0.8} (Pr)^{n}$

$n = 0.4$ for heating process

$n = 0.3$ for cooling process

This equation is valid for

$0.6 < Pr < 160$; $Re > 10{,}000$; $\frac{L}{D} > 60$

Also, $Nu = 0.036 (Re)^{0.8} (Pr)^{0.33} \left(\frac{D}{L}\right)^{0.055}$

This equation is valid for

$10 < \frac{L}{D} < 400$; $Re < 10{,}000$

5. Equivalent diameter for rectangular section,

$$D_h \text{ (or) } D_e = \frac{4A}{P} = \frac{4(L \times W)}{2(L+W)}$$

Where, A - Area in m^2

P - Perimeter in m

L - Length in m

W - width in m

6. Equivalent diameter for hollow cylinder

$$D_h \text{ (or) } D_e = \frac{4A}{P} = \frac{4 \times \frac{\pi}{4}\left[D_o^2 - D_i^2\right]}{\pi\left[D_o + D_i\right]}$$

Where D_o - outer diameter

D_i - Inner diameter

7. Heat transfer

$$Q = hA(T_w - T_\infty)$$

Where, $A = \pi D L$

Also, $Q = m \cdot c_p (T_{mo} - T_{mi})$

8. Mass flow rate, $\dot{m} = \rho A U$, Kg/s

Where, ρ - Density in Kg/m^3

A - Area in m^2 $A = \frac{\pi}{4} D^2$

U - velocity in m/s

4. 205 Kg/hr of air are cooled from 100°C to 30°C by flowing through a 3.5 cm inner diameter pipe coil bent into a helix of 0.6 m diameter. Calculate the value of air side heat transfer coefficient, if the properties of air at 65°C are $K = 0.0298$ W/mK, $\mu = 0.003$ Kg/hr-m, $Pr = 0.7$, $\rho = 1.044$ Kg/m^3

Given:

Mass flow rate, $\dot{m} = 205$ Kg/hr $= \frac{205}{3600}$ Kg/s

$\dot{m} = 0.056$ Kg/s

Inlet temp. of air, $T_{mi} = 100°C$

Outlet temp. of air, $T_{mo} = 30°C$

Diameter, $D = 3.5\,cm = 0.035\,m$

Mean temp., $T_m = \frac{T_{mi} + T_{mo}}{2} = \frac{100+30}{2} = 65°C$

To find:

Heat transfer coefficient (h)

Solution:

Kinematic viscosity, $\nu = \frac{\mu}{\rho} = \frac{0.003/3600}{1.044}$

$\mu = \frac{0.003}{3600}$ Kg/s-m $\quad \nu = 7.98 \times 10^{-7}$ m^2/s

Mass flow rate, $\dot{m} = \rho A U \Rightarrow U = \frac{\dot{m}}{\rho A}$

$$U = \frac{0.056}{1.044 \times \frac{\pi}{4}(0.035)^2}$$

$U = 55.7$ m/s

Reynold's Number, $Re = \frac{UD}{\nu} = \frac{55.7 \times 0.035}{7.98 \times 10^{-7}}$

$$Re = 2.44 \times 10^6 > 2300$$

$\therefore$ The flow is turbulent

From HMTDB P125, For turbulent flow ($Re > 10{,}000$)

$$Nu = 0.023 \times (Re)^{0.8} \times (Pr)^n \quad (n = 0.3 \text{ for cooling})$$

$$Nu = 0.023 \times (2.44 \times 10^6)^{0.8} \times (0.7)^{0.3}$$

$$Nu = 2661.7$$

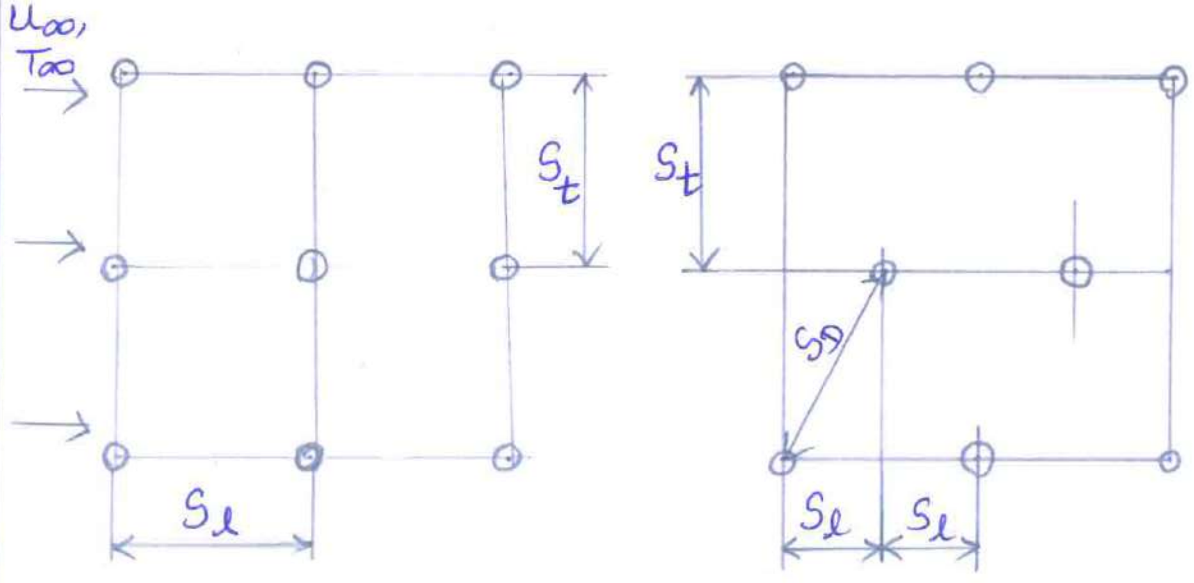
Also, $Nu = \frac{hD}{K} \Rightarrow h = \frac{Nu \times K}{D}$

Heat transfer co-efficient, $h = \frac{2661.7 \times 0.0298}{0.035}$

$$h = 2266.2 \ W/m^2K$$

Flow over bank of tubes

The tube rows of a bank may be either Staggered or inline



(a) inline (b) staggered

Formulae

1. Max. Velocity, $U_{max} = U \times \frac{S_t}{S_t - D}$

where S_t - Transverse pitch in m

2. Reynolds Number, $Re = \frac{U_{max} \times D}{\nu}$

3. Nusselt Number, $Nu = 1.13 \times (Pr)^{0.333} \left[C . Re^n\right]$

5. In a surface condenser, water flows through staggered tubes while the air is passed in cross flow over the tubes. The temp. and velocity of air are 30°c and 8 m/s respectively. The longitudinal and transverse pitches are 22 mm and 20 mm respectively. The tube outside dia. is 18 mm and tube surface temp. is 90°c. Calculate the heat transfer coefficient.

Given:

Fluid temp., $T_\infty = 30°c$

Velocity, $U = 8$ m/s

Longi. Pitch, $S_l = 22$ mm $= 22 \times 10^{-3}$ m

Trans. Pitch, $S_t = 20$ mm $= 20 \times 10^{-3}$ m

Diameter, $D = 18$ mm $= 18 \times 10^{-3}$ m

Tube surface temp., $T_w = 90°c$

To find:

Heat transfer coefficient, h

Solution:

Film temp., $T_f = \frac{T_w + T_\infty}{2} = \frac{90 + 30}{2} = 60°c$

From HMTDB, P34, properties of air at 60°C

$\rho = 1.060\ Kg/m^3$, $\nu = 18.97 \times 10^{-6}\ m^2/s$

$Pr = 0.696$, $K = 0.02896\ W/mK$

W.K.T., Max. velocity, $U_{max} = U \times \frac{S_t}{S_t - D}$

$$U_{max} = 8 \times \frac{0.020}{0.020 - 0.018} = 80\ m/s$$

Reynold's Number, $Re = \frac{U_{max} \times D}{\nu} = \frac{80 \times 0.018}{18.97 \times 10^{-6}} = 7.5 \times 10^4$

$\frac{S_t}{D} = \frac{0.020}{0.018} = 1.11$, $\frac{S_l}{D} = \frac{0.022}{0.018} = 1.22$

From HMTDB, P123

corresponding c and n values are,

$C = 0.518$ and $n = 0.556$

Nusselt Number, $Nu = 1.13\,(Pr)^{0.333}\left[C\,(Re)^n\right]$

$$Nu = 1.13 \times (0.696)^{0.333} \times \left[0.518 \times (7.5 \times 10^4)^{0.556}\right]$$

$Nu = 266.3$

Also, $Nu = \frac{hD}{K}$

$$\Rightarrow \quad h = \frac{Nu \times K}{D}$$

$$h = \frac{266.3 \times 0.02896}{0.018}$$

Heat transfer coefficient, $h = 428.6\ W/m^2K$

External flow over cylinders

Formulae:

1. Film temperature, $T_f = \frac{T_w + T_\infty}{2}$

2. $Re = \frac{UD}{\nu}$

From HMTDB, P 115

3. $Nu = C.(Re)^m.(Pr)^{0.333}$

Also $Nu = \frac{hD}{K}$

4. $Q = hA(T_W - T_\infty)$

$A = \pi D L$

For sphere, Page 119

3. $Nu = 0.37(Re)^{0.6}$

4. $Q = hA(T_W - T_\infty)$

$A = 4\pi r^2$

6 Air at 15°C, 30 km/hr flows over a cylinder of 400 mm diameter and 1500 mm height with surface temp. of 45°C. Calculate the heat loss.

Given:

Temp. of air, $T_\infty = 15°C$

Velocity of air, $U = 30\ Km/hr = \frac{30 \times 10^3}{3600} = 8.33\ m/s$

Cylinder dia, $D = 400\ mm = 0.4\ m$

Cylinder height, $L = 1500\ mm = 1.5\ m$

Surface temp., $T_w = 45°C$

Solution:-

Film temp., $T_f = \frac{T_w + T_\infty}{2} = \frac{45+15}{2} = 30°C$

From HMTDB P33, properties of air at 30°C

$\rho = 1.165\ Kg/m^3$ $\quad$ $Pr = 0.701$

$\nu = 16 \times 10^{-6}\ m^2/s$ $\quad$ $K = 0.02675\ W/mK$

$$Re = \frac{UD}{\nu} = 2.08 \times 10^5$$

From HMTDB P115

$$Nu = C.(Re)^m.(Pr)^{0.333}$$

Re_D value is 2.08×10^5

Corresponding $c = 0.0266$ and $m = 0.805$

$$Nu = 0.0266 \times (2.08 \times 10^5)^{0.805} \times (0.701)^{0.333}$$

$$Nu = 451.3$$

Also, $Nu = \frac{hD}{K} \Rightarrow h = \frac{Nu \times K}{D} = \frac{451.3 \times 0.02675}{0.4}$

$$h = 30.18 \ W/m^2K$$

$$Q = hA(T_w - T_\infty) = h \times \pi DL \times (T_w - T_\infty)$$

$$Q = 30.18 \times \pi \times 0.4 \times 1.5 \times (45 - 15)$$

Heat loss, $Q = 1706.6$ Watts

7. Air at 40°C flows over a tube of 6 cm diameter with a velocity of 30 m/s. The tube surface temp. is 120°C. Calculate the heat transfer coefficient.

Given:-

Temp. of air, $T_\infty = 40°C$

Tube diameter, $D = 6 cm = 0.06 m$

Surface temp., $T_w = 120°C$

Velocity of air, $U = 30 m/s$

To find:

Heat transfer coefficient

Solution:

Film temperature, $T_f = \frac{T_w + T_\infty}{2} = \frac{120+40}{2} = 80°C$

From HMTDB P33, properties of air at 80°C

$\rho = 1\ Kg/m^3$ $\qquad \nu = 21.09 \times 10^{-6}\ m^2/s$

$Pr = 0.692$ $\qquad K = 0.03047\ W/mK$

$R_e = \frac{UD}{\nu} = \frac{30 \times 0.06}{21.09 \times 10^{-6}} = 0.853 \times 10^5$

From HMTDB P115

$Nu = C\,(Re_D)^m\,(Pr)^{0.333}$

Re value is 0.853×10^5

Corresponding $C = 0.0266$, $m = 0.805$

$Nu = 0.0266 \times (0.853 \times 10^5)^{0.805} \times (0.692)^{0.333}$

$Nu = 219.3$

Also, $Nu = \frac{hD}{K} \Rightarrow h = \frac{Nu \times K}{D}$

$h = \frac{219.3 \times 0.03047}{0.06}$

Heat transfer coefficient, $h = 111.3\ W/m^2K$

Internal flow through tubes

8. Engine oil flows through a 50 mm dia. tube at an average temp. of 147°C. The flow velocity is 80 cm/s. Calculate the average heat transfer coefficient if the tube wall is maintained at a temperature of 200°C and it is 2m long.

Given:

Tube Diameter, $D = 50\,mm = 0.05\,m$

Average temperature, $T_m = 147°C$

Flow velocity, $U = 80\,cm/s = 0.80\,m/s$

Tube wall temp., $T_W = 200°C$

Length of the tube, $L = 2\,m$

To find:

Heat transfer coefficient

Solution: From HMT DB P24

properties of engine oil at 147°C

$\rho = 816\ Kg/m^3$ $\qquad$ $Pr = 116$

$\nu = 8 \times 10^{-6}\ m^2/s$ $\qquad$ $K = 0.1338\ W/mK$

$$Re = \frac{UD}{\nu} = \frac{0.8 \times 0.05}{8 \times 10^{-6}} = 5000$$

Here $Re > 2300$ ∴ The flow is turbulent

$$\frac{L}{D} = \frac{2}{0.05} = 40$$

$$10 < \frac{L}{D} < 400$$

From HMT DB P125, For turbulent flow, $Re < 10{,}000$

$$\therefore Nu = 0.036\,(Re)^{0.8}(Pr)^{0.33}\left(\frac{D}{L}\right)^{0.055}$$

$$Nu = 0.036\,(5000)^{0.8} \times (116)^{0.33} \times \left(\frac{0.05}{2}\right)^{0.055}$$

$$Nu = 128.42$$

Also, $Nu = \frac{hD}{K} \Rightarrow h = \frac{Nu \times K}{D} = \frac{128.42 \times 0.1338}{0.05}$

Heat transfer coefficient, $h = 343.65\ W/m^2K$

9. A system for heating water from an inlet temperature of 20°c to an outlet temperature of 40°c involves passing the water through a 2.5 cm diameter steel pipe. The pipe surface temp. is maintained at 110°c by condensing steam on its surface. For a water mass flow rate of 0.5 Kg/min, find the length of the tube desired.

Given:

Inlet temperature, $T_{mi} = 20°c$

Outlet temperature, $T_{mo} = 40°c$

Steel pipe diameter, $D = 2.5 cm = 0.025 m$

Pipe surface temp., $T_w = 110°c$

Mass flow rate, $\dot{m} = 0.5 \text{ Kg/min} = \frac{0.5}{60} \text{ Kg/s}$

$\dot{m} = 8.33 \times 10^{-3} \text{ Kg/s}$

To find:

Length of the tube (L)

Solution:

Bulk mean temp., $T_m = \frac{T_{mi} + T_{mo}}{2} = \frac{20+40}{2} = 30°c$

Properties of water at 30°c

$\rho = 997 \text{ Kg/m}^3$

$\nu = 0.857 \times 10^{-6} \text{ m}^2/\text{s}$

$Pr = 5.5$

$K = 0.610 \text{ W/mK}$

$C_p = 4178 \text{ J/KgK}$

W.K.T. $\dot{m} = \rho A U$

$$\dot{m} = \rho \times \frac{\pi}{4} D^2 \times U$$

$$U = \frac{\dot{m}}{\rho \times \frac{\pi}{4} D^2} = \frac{8.33 \times 10^{-3}}{997 \times \frac{\pi}{4}(0.025)^2}$$

$$U = 0.017 \text{ m/s}$$

$$Re = \frac{UD}{\nu} = \frac{0.017 \times 0.025}{0.857 \times 10^{-6}} = 495 < 2300$$

∴ The flow is laminar.

From HMTDB P123, For laminar flow

$$Nu = 3.66$$

Also, $Nu = \frac{hD}{K} \Rightarrow h = \frac{Nu \times K}{D} = \frac{3.66 \times 0.610}{0.025}$

Heat transfer coefficient, $h = 89.3 \text{ W/m}^2\text{K}$

Heat transfer, $Q = \dot{m} C_p \Delta T = \dot{m} C_p (T_{mo} - T_{mi})$

$$= 8.33 \times 10^{-3} \times 4178 \times (40 - 20)$$

$$Q = 695 \text{ W}$$

Also, $Q = hA(T_w - T_m)$

$$Q = h \times \pi D L (T_w - T_m)$$

$$\therefore L = \frac{Q}{h \times \pi \times D \times (T_w - T_m)}$$

$$L = \frac{695}{89.3 \times \pi \times 0.025 \times (110 - 30)}$$

Length of the tube, $L = 1.24 \text{ m}$

Free convection - Formulae

1. Film temperature, $T_f = \frac{T_w + T_\infty}{2}$

2. Coefficient of thermal expansion, $\beta = \frac{1}{T_f}$, K^{-1}

3. Nusselt Number, $Nu = \frac{hL}{K}$

Vertical plate, HMTDB P134

4. Grashof Number, $Gr = \frac{g \times \beta \times L^3 \times \Delta T}{\nu^2}$

5. $Gr\,Pr < 10^9 \rightarrow$ laminar flow

   $Gr\,Pr > 10^9 \rightarrow$ Turbulent flow

6. For laminar flow, vertical plate (HMTDB P135)

   $Nu = 0.59\,(Gr\,Pr)^{0.25}$

7. For turbulent flow, vertical plate

   $Nu = 0.10\,[Gr\,Pr]^{0.333}$

8. Heat transfer, $Q = hA(T_w - T_\infty)$

Horizontal plate, HMTDB P135

9. Grashof Number, $Gr = \frac{g \times \beta \times L_c^3 \times \Delta T}{\nu^2}$

   where L_c - characteristic length $= \frac{W}{2}$

   W - width of the plate

10. For Horizontal plate, upper surface heated

    $Nu = 0.54\,(Gr\,Pr)^{0.25}$, if $2 \times 10^4 < Gr\,Pr < 8 \times 10^6$

    $Nu = 0.15\,(Gr\,Pr)^{0.333}$, if $8 \times 10^6 < Gr\,Pr < 10^{11}$

11. For Horizontal plate, lower surface heated

    $Nu = 0.27\,[Gr\,Pr]^{0.25}$, if $10^5 < Gr\,Pr < 10^{11}$

12. Heat transfer

$Q = (h_u + h_l) \times A \times (T_w - T_\infty)$

h_u - Upper surface heated heat transfer co-efficient, W/m^2K

h_l - Lower surface heated heat transfer coefficient, W/m^2K

Horizontal cylinder [HMTDB P137]

13. $Nu = C(Gr\,Pr)^m$

14. $Q = hA(T_w - T_\infty)$, where $A = \pi DL$

10. A large vertical plate 5m height is maintained at 100°c and exposed to air at 30°C. Calculate the convective heat transfer coefficient.

Given:-

Height of the vertical plate, $L = 5\,m$

Surface temperature, $T_w = 100°c$

Air temperature, $T_\infty = 30°c$

To find:

Heat transfer coefficient, h

Solution: From HMTDB P33, properties of air 65°c

Film temp., $T_f = \dfrac{T_w + T_\infty}{2} = 65°C$

$\rho = 1.0445\ Kg/m^3$

$\nu = 19.495 \times 10^{-6}\ m^2/s$

$Pr = 0.695$

$K = 0.02931\ W/mK$

$$\beta = \frac{1}{T_f} = \frac{1}{65+273} = \frac{1}{338} = 2.958 \times 10^{-3}\ K^{-1}$$

From HMT DB, P134

$$Gr = \frac{g \times \beta \times L^3 \times \Delta T}{\nu^2}$$

$$Gr = \frac{9.81 \times 2.958 \times 10^{-3} \times 5^3 \times (100-30)}{(19.495 \times 10^{-6})^2}$$

$$Gr = 6.68 \times 10^{11}$$

$$Gr\,Pr = 6.68 \times 10^{11} \times 0.695 = 4.64 \times 10^{11} > 10^9$$

∴ The flow is turbulent.

From HMT DB, P135

$$Nu = 0.10\,(Gr\,Pr)^{0.333}$$

$$Nu = 0.10 \times (4.64 \times 10^{11})^{0.333}$$

$$Nu = 767.27$$

Also, $Nu = \frac{hL}{K} \Rightarrow h = \frac{Nu \times K}{L} = \frac{767.27 \times 0.02931}{5}$

Heat transfer coefficient, $h = 4.49\ W/m^2K$

11. A steam pipe 10 cm outside diameter runs horizontally in a room at 23°C. Take the outside surface temperature of pipe as 165°C. Determine the heat loss per metre length of the pipe.

Given:

Pipe diameter, $D = 10\,cm = 0.1\,m$

Room temperature, $T_\infty = 23°C$

Surface temperature, $T_w = 165°C$

To find: Heat loss per metre length

Solution:-

$$\text{Film temp., } T_f = \frac{T_w + T_\infty}{2} = \frac{165+23}{2} = 94°C$$

$$T_f \simeq 95°C$$

From HMT DB, P34, properties of air at 95°C

$\rho = 0.959\ Kg/m^3$

$\nu = 22.615 \times 10^{-6}\ m^2/s$

$Pr = 0.689$

$K = 0.03169\ W/mK$

$$\beta = \frac{1}{T_f} = \frac{1}{94+273} = 2.72 \times 10^{-3}\ K^{-1}$$

From HMT DB, P134

$$Gr = \frac{g \times \beta \times D^3 \times \Delta T}{\nu^2}$$

$$Gr = \frac{9.81 \times 2.72 \times 10^{-3} \times (0.1)^3 \times (165-23)}{(22.615 \times 10^{-6})^2}$$

$$Gr = 7.4 \times 10^6$$

$$GrPr = 7.4 \times 10^6 \times 0.689 = 5.09 \times 10^6$$

From HMT DB, P137

$$Nu = C\,[GrPr]^m$$

For GrPr value 5.09×10^6,

Corresponding $c = 0.48$ and $m = 0.25$

$$Nu = 0.48\,[5.09 \times 10^6]^{0.25}$$

$$Nu = 22.79$$

$$\text{Also, } Nu = \frac{hD}{K} \Rightarrow h = \frac{Nu \times K}{D} = \frac{22.79 \times 0.03169}{0.1}$$

$$h = 7.22\ W/m^2K$$

$$Q = hA\Delta T$$

$$Q = h \times \pi D L (T_w - T_\infty) \qquad [\because L = 1m]$$

Heat loss per metre length

$$\frac{Q}{L} = h \times \pi D (T_w - T_\infty)$$

$$Q/L = 7.22 \times \pi \times 0.1 \times (165-23)$$

$$Q/L = 322.08 \text{ W/m} \qquad \left[Q/L = Q/1 = Q\right]$$

$$Q = 322.08 \text{ W/m}$$

Boiling:- The change of phase from liquid to vapour state is known as boiling.

Condensation: The change of phase from vapour to liquid state is known as condensation.

Applications:-

1. Thermal and Nuclear power plant
2. Refrigerating systems
3. Air conditioning systems
4. Process of heating and cooling
5. Heating of metal in furnaces

Boiling heat transfer phenomena

Boiling is a convection process involving a change of phase from liquid to vapour state. This is possible only when the temp. of the surface (T_w) exceeds the saturation temp. of liquid (T_{sat}).

$$Q = hA(T_w - T_{sat}) = hA\Delta T$$

where $\Delta T = (T_w - T_{sat})$ is known as excess temperature

If heat is added to a liquid from a submerged solid surface, the boiling process is referred to as pool boiling. In this case the liquid above the hot surface is essentially stagnant and its motion near the surface is due to free convection and mixing induced by bubble growth and detachment.

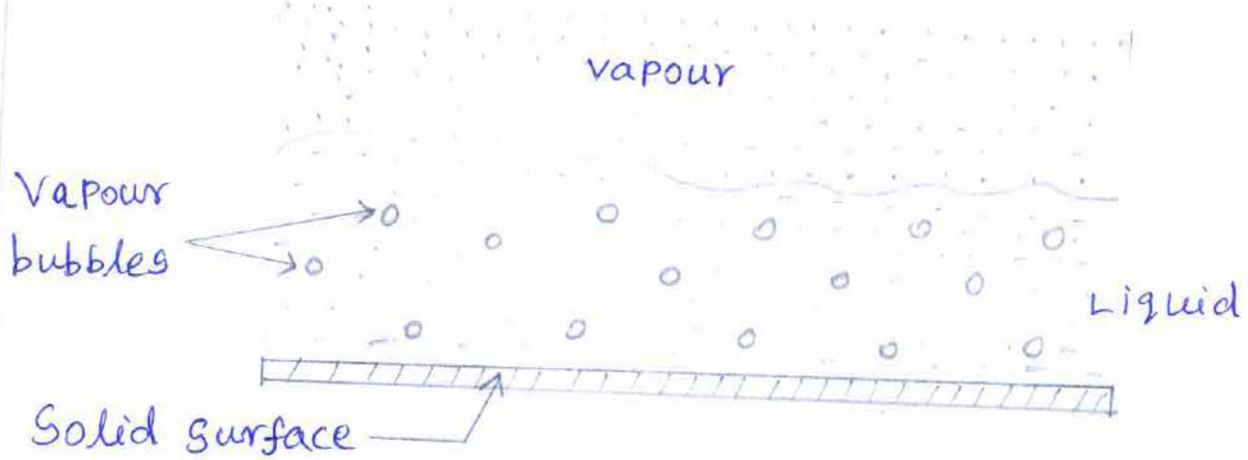
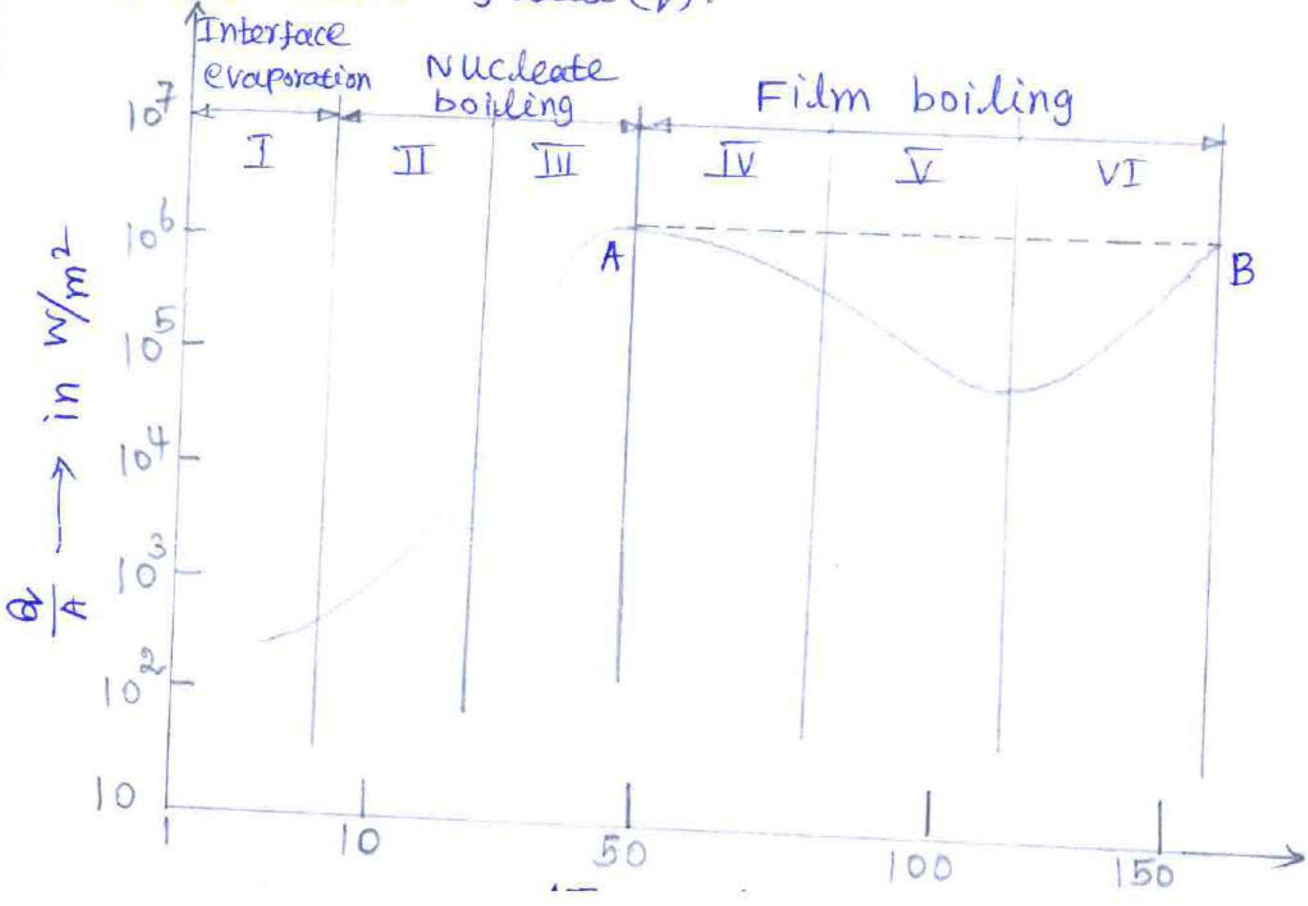


Fig. 1. Pool boiling

Fig. 1. shows the temp. distribution in Saturated Pool boiling with a liquid vapour interface.

The different regions of boiling are indicated in fig. 2. This specific curve obtained from an electrically heated Platinum wire submerged in a pool of water by varying its surface temperature and measuring the surface heat flux (q).



I - Free convection
II - Bubbles condense in super heated liquid
III - Bubbles raise to surface
IV - Unstable film
V - Stable film
VI - Radiation coming into play

Fig.2. Pool boiling curve for water

1. Interface evaporation:

Evaporation process with no bubble formation exists in region I. In this region the excess temp. ΔT is very small (5°c). Here the liquid near the surface is superheated slightly, and evaporation takesplace at the liquid surface.

2. Nucleate boiling

This type of boiling exists in regions II and III. The nucleate boiling begins at region II. As the excess temp. is further increased, bubbles are formed more rapidly and rapid evaporation takesplace. This is indicated in region III. Nucleate boiling exists upto $\Delta T = 50°c$. The maximum heat flux, known as critical heat flux, occurs at point A.

3. Film boiling:

Film boiling exists in regions IV, V and VI.

In region IV the vapour film is not stable and collapses and reforms rapidly. With further increase in ΔT, the vapour film is stabilised as indicated in region V.

The surface temp. required to maintain a stable film are high and under these conditions some amount of heat is lost by the surface due to radiation. This is indicated in VI.

From Fig. 2. shows, it is clear that high heat transfer rates are associated with small values of the excess temperature in nucleate boiling region.

Flow boiling

Flow boiling or forced convection boiling may occur when a fluid is forced through a pipe or over a surface which is maintained at a temperature higher than the saturation temperature of the fluid. This type of boiling occurs in water tube boilers involving forced convection.

Boiling correlations:

It is obvious from the boiling curve that various physical mechanisms are involved in different regions and there will be many types of correlations for the boiling process. Some of them are given below

1. Nucleate pool boiling

From HMT DB, P142

a. Heat flux $\left[q = \frac{Q}{A}\right]$

$$\frac{Q}{A} = \mu_l \, h_{fg} \left[\frac{g \times (\rho_l - \rho_v)}{\sigma}\right]^{0.5} \times \left[\frac{C_{pl} \times \Delta T}{C_{sf} \times h_{fg} \, P_r^{\,n}}\right]^3 \quad \text{—} ①$$

where, $\frac{Q}{A} = q$ = heat flux, W/m^2

μ_l - Dynamic viscosity of liquid, $\frac{Ns}{m^2}$

h_{fg} - enthalpy of evaporation, J/Kg

g - Acceleration due to gravity, $9.81 \, m/s^2$

ρ_l - Density of liquid, Kg/m^3

ρ_v - Density of vapour, Kg/m^3

σ - Surface tension for liquid vapour interface, N/m

C_{pl} - Specific heat of liquid, J/KgK

C_{sf} - Surface fluid constant

P_r - Prandtl Number

ΔT - Excess temperature $= T_w - T_{sat}$, °C

n = 1 for water & $n = 1.7$ for other fluids

b. Critical heat flux

$$\frac{Q}{A} = 0.18 \, h_{fg} \, \rho_v \left[\frac{\sigma \times g \, (\rho_l - \rho_v)}{\rho \, v^2}\right]^{0.25} \quad \text{—} ②$$

c. Heat transfer

$$Q = m \times h_{fg} \quad \text{—} ③$$

2. Film Pool boiling

From HMT DB P142

a. Heat transfer coefficient

$$h = h_{conv} + 0.75\, h_{rad} \quad \text{--- (4)}$$

$$h_{conv} = 0.62 \left[\frac{K_v^3 \times \rho_v \times (\rho_l - \rho_v) \times g \times (h_{fg} + 0.4\, C_{pv} \Delta T)}{\mu_v D \Delta T}\right]^{0.25} \quad \text{--- (5)}$$

where, K_v - Thermal conductivity of vapour, W/mK

ρ_v - Density of vapour, Kg/m^3

ρ_l - Density of liquid, Kg/m^3

g - Acceleration due to gravity, $9.81\, m/s^2$

h_{fg} - Enthalpy of evaporation, J/Kg

C_{pv} - Specific heat of water at const. Pressure

μ_v - Dynamic viscosity of vapour, NS/m^2

D - Diameter, m

ΔT - Excess temperature $= T_w - T_{sat}$, in °c

$$h_{rad} = \sigma \epsilon \left[\frac{T_w^4 - T_{sat}^4}{T_w - T_{sat}}\right] \quad \text{--- (6)}$$

where σ - Stefen Boltzmann constant

$\sigma = 5.67 \times 10^{-8}\, W/m^2K^4$

ϵ - emissivity

T_w - surface temperature, °c

T_{sat} - Saturation temperature, °c

b. Excess temperature

$\Delta T = T_w - T_{sat} > 50°c$ for film pool boiling

Problem:

1. An aluminium pan of 15 cm diameter is used to boil water and the water depth at the time of boiling is 2.5 cm. The pan is placed on an electric stove and the heating element raises the temperature of the pan to 110°C. Calculate the power input for boiling and the rate of evaporation. Take $C_{sf} = 0.0132$

Given:

Diameter, $d = 15\ cm = 0.15\ m$

Distance, $x = 2.5\ cm = 0.025\ m$

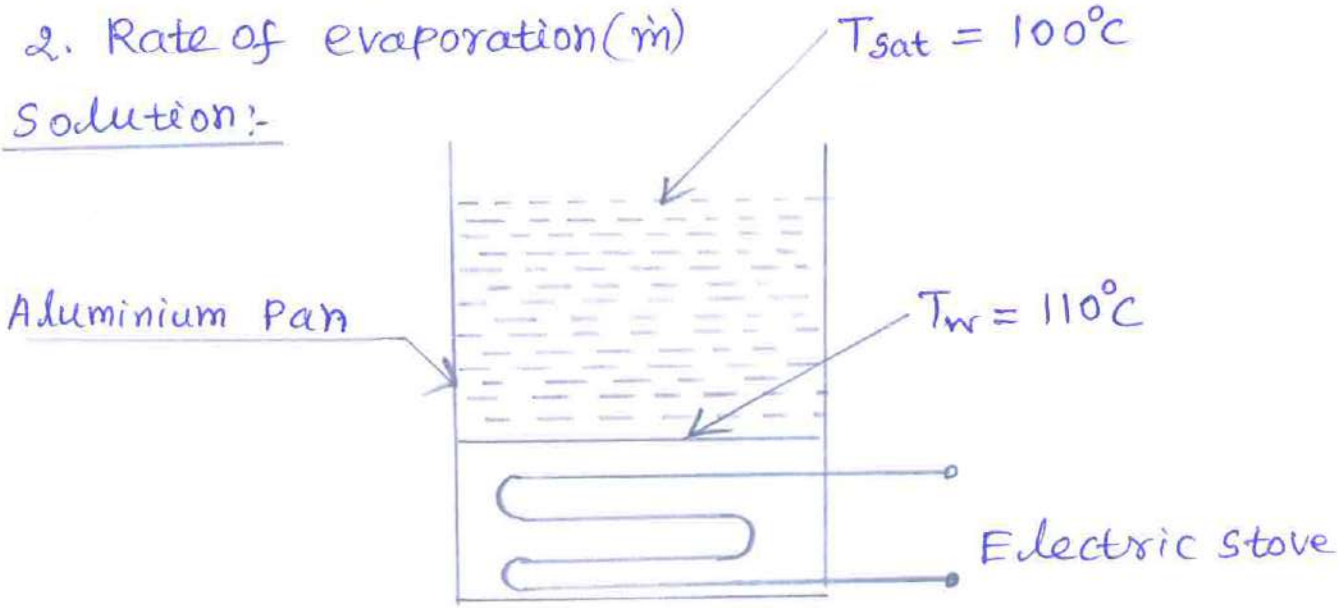
Surface temp., $T_w = 110°C$

$C_{sf} = 0.0132$

To find:

1. Power input (P)
2. Rate of evaporation ($\dot{m}$)

Solution:-



W.K.T. Saturation temp. of water is 100°C

$T_{sat} = 100°C$

Properties of water at 100°C [From HMTDB P21]

Density, $\rho_l = 961\ kg/m^3$

Kinematic viscosity, $\nu = 0.293 \times 10^{-6}\ m^2/s$

Prandtl Number, $P_r = 1.740$

Specific heat, $C_{pl} = 4216\ J/KgK$

Dynamic viscosity, $\mu_l = \rho_l \times \nu$

$= 961 \times 0.293 \times 10^{-6}$

$= 281.57 \times 10^{-6}\ NS/m^2$

From steam table at $100°c$

Enthalpy of evaporation, $h_{fg} = 2256.9\ KJ/Kg$

$= 2256.9 \times 10^3\ J/Kg$

Specific volume of vapour, $V_g = 1.673\ m^3/Kg$

Density of vapour, $\rho_v = \frac{1}{V_g} = \frac{1}{1.673}$

$\rho_v = 0.597\ Kg/m^3$

$\Delta T = T_w - T_{sat} = 110 - 100 = 10°c$

$\Delta T = 10°c < 50°c$. So, this is Nucleate Pool boiling

1. Power input for boiling

Heat flux, $\frac{Q}{A} = \mu_l \times h_{fg}\left[\frac{g \times (\rho_l - \rho_v)}{\sigma}\right]^{0.5} \times \left[\frac{C_{pl} \times \Delta T}{C_{sf} \times h_{fg}\, P_r^{\,n}}\right]^3$ — (1)

[From HMT DB P142]

Where, $n = 1$ for water

σ = Surface tension for liquid vapour interface

From HMTDB P144

At $100°c$, $\sigma = 0.0588\ N/m$

Substitute all the values in eqn (1)

$$\frac{Q}{A} = 281.57 \times 10^{-6} \times 2256.9 \times 10^3 \times \left[\frac{9.81(961 - 0.597)}{0.0588}\right]^{0.5}$$

$$* \left[\frac{4216 \times 10}{0.013 \times 2256.9 \times 10^3 \times 1.740}\right]^3$$

$\frac{Q}{A} = 1.43 \times 10^5 \ W/m^2$ $\qquad [\because A = \frac{\pi}{4}d^2]$

Heat transfer, $Q = 1.43 \times 10^5 \times \frac{\pi}{4}(0.15)^2$

$Q = 2527$ watts

Power input for boiling, $P = 2527$ watts

2. Rate of evaporation ($\dot{m}$)

Heat transferred, $Q = \dot{m} \times h_{fg}$

$\therefore \dot{m} = \frac{Q}{h_{fg}} = \frac{2527}{2256.9 \times 10^3} = 1.11 \times 10^{-3}$ Kg/s

Result:-

1. Power $= 2527$ W
2. Rate of evaporation, $\dot{m} = 1.11 \times 10^{-3}$ Kg/s

2. Water is boiled at the rate of 24 Kg/hour in a polished copper pan, 300 mm in diameter, at atmospheric pressure. Assuming nucleate boiling conditions. Calculate the temperature of the bottom surface of the pan.

Given:

Mass flow rate, $\dot{m} = 24$ Kg/hr $= \frac{24}{3600}$ Kg/s

$\dot{m} = 6.6 \times 10^{-3}$ Kg/s

Diameter, $d = 300$ mm $= 0.3$ m

To find:

Surface temperature, T_w

Solution:-

W.K.T. Saturation temp. of water is 100°C

$T_{sat} = 100°C$

Properties of water at 100°C, [From HMT DB P21]

Density, $\rho_l = 961\ Kg/m^3$

Kinematic viscosity $\nu = 0.293 \times 10^{-6} m^2/s$

$P_r = 1.740$

$C_{pl} = 4216\ J/KgK$

Dynamic viscosity $\mu_l = \rho_l \times \nu = 961 \times 0.293 \times 10^{-6}$

$\mu_l = 281.57 \times 10^{-6}\ NS/m^2$

From steam table at 100°C

Enthalpy of Evaporation, $h_{fg} = 2256.9\ KJ/Kg$

$= 2256.9 \times 10^3\ J/Kg$

Specific volume of vapour, $V_g = 1.673\ m^3/Kg$

Density of vapour, $\rho_v = \frac{1}{V_g} = \frac{1}{1.673} = 0.597\ Kg/m^3$

For Nucleate boiling

Heat flux, From HMT DB P142

$$\frac{Q}{A} = \mu_l \times h_{fg}\left[\frac{g\times(\rho_l - \rho_v)}{\sigma}\right]^{0.5} \times \left[\frac{C_{pl}\times \Delta T}{C_{sf}\times h_{fg}\, P_r^{n}}\right]^3 \quad \text{--- (1)}$$

Heat transferred, $Q = \dot{m} \times h_{fg}$

$$\frac{Q}{A} = \frac{\dot{m}\times h_{fg}}{A} = \frac{6.6\times 10^{-3} \times 2256.9\times 10^3}{\frac{\pi}{4}\times(0.3)^2}$$

$$\frac{Q}{A} = 210 \times 10^3\ W/m^2$$

From HMTDB, P144, at 100°C

$\sigma = 0.0588\ N/m$

For water - copper $\Rightarrow C_{sf} = 0.013$ [P143]

$n = 1$ for water

Substitute all the values in eqn ①.

$$210 \times 10^3 = 281.57 \times 10^{-6} \times 2256.9 \times 10^3 \times \left[\frac{9.81 \times (961 - 0.597)}{0.0588}\right]^{0.5} \times \left[\frac{4216 \times \Delta T}{0.013 \times 2256.9 \times 10^3 \times 1.74}\right]^3$$

$$\left[\frac{4216 \times \Delta T}{51051.1}\right]^3 = 0.825$$

$$\left[\frac{4216 \times \Delta T}{51051.1}\right]^{\cancel{3} \times \frac{1}{\cancel{3}}} = (0.825)^{1/3}$$

$$0.0825\ \Delta T = 0.937$$

$$\Delta T = 11.35°C$$

Also, $\Delta T = T_w - T_{sat}$

$$11.35 = T_w - 100$$

$$T_w = 111.35°C$$

Result:

Surface temperature, $T_w = 111.35°C$

Condensation:

The change of phase from vapour to liquid state is known as condensation.

Modes of condensation:

1. Film wise condensation
2. Dropwise condensation

Filmwise condensation

The liquid condensate wets the solid surface, spreads out and forms a continuous film over the entire surface is known as filmwise condensation.

Film condensation occurs when a vapour free from impurities.

Dropwise condensation

In dropwise condensation, the vapour condenses into small liquid droplets of various sizes which fall down the surface in a random fashion.

Heat transfer rates in dropwise condensation may be as much as 10 times higher than in filmwise condensation.

Correlation for filmwise condensing process

From HMT DB, P 148

a. Filmwise thickness for laminar flow vertical surface

$$\delta_x = \left[\frac{4 \mu K x (T_{sat} - T_w)}{g \times h_{fg} \times \rho^2}\right]^{0.25} \quad \text{--- (1)}$$

b. Local heat transfer coefficient (h_x) for vertical surface, laminar flow

$$h_x = \frac{K}{\delta_x} \quad \text{—— ②}$$

C. Average heat transfer coefficient (h) for vertical surface, laminar flow

$$h = 0.943 \left[\frac{K^3 \rho^2 g\ h_{fg}}{\mu L\ (T_{sat} - T_W)}\right]^{0.25} \quad \text{—— ③}$$

The factor 0.943 may be replaced by 1.13 for more accurate result as suggested Mc adams

$$h = 1.13 \left[\frac{K^3 \rho^2 g\ h_{fg}}{\mu L\ (T_{sat} - T_W)}\right]^{0.25} \quad \text{—— ④}$$

D. Average heat transfer coefficient for Horizontal surface, laminar flow

$$h = 0.728 \left[\frac{K^3 \rho^2 g\ h_{fg}}{\mu D (T_{sat} - T_W)}\right]^{0.25} \quad \text{—— ⑤}$$

E. Average heat transfer coefficient for bank of tubes, laminar flow

$$h = 0.728 \left[\frac{K^3 \rho^2 g\ h_{fg}}{\mu\ N D (T_{sat} - T_W)}\right]^{0.25} \quad \text{—— ⑥}$$

F. For laminar flow, $Re \leq 1800$

$$Re = \frac{4 \dot{m}}{P \mu}$$, where P – Perimeter

G. For turbulent flow, $Re > 1800$

H. Average heat transfer coefficient for vertical surface, turbulent flow

$$h = 0.0077 (Re)^{0.4} \left[\frac{K^3 \rho^2 g}{\mu^2}\right]^{0.333} \quad \text{—— ⑦}$$

3. Dry Saturated steam at a pressure of 2.45 bar condenses on the surface of a vertical tube of height 1m. The tube surface temperature is kept at 117°C. Estimate the thickness of the condensate film.

Given:-

Pressure, $P = 2.45$ bar

Distance (or) height, $x = 1$ m

Surface temp., $T_w = 117°C$

To find:

Thickness of the condensate film, δ_x

Solution:

From steam Table P10 (R.S. Khurmi)

Properties of steam at 2.45 bar

$T_{sat} = 127°C$

$h_{fg} = 2183$ KJ/Kg

$h_{fg} = 2183 \times 10^3$ J/Kg

W.K.T. Film temp., $T_f = \frac{T_w + T_{sat}}{2} = \frac{117+127}{2}$

$T_f = 122°C$

From HMT DB P22, properties of water at 120°C

$\rho = 945$ Kg/m³

$\nu = 0.247 \times 10^{-6}$ m²/s

$K = 0.685$ W/mK

$\mu = \rho \times \nu = 945 \times 0.247 \times 10^{-6} = 2.33 \times 10^{-4} \frac{N \cdot s}{m^2}$

Assume film is laminar.

For vertical surface,

$$\delta_x = \left[\frac{4\,\mu K x\,(T_{sat}-T_w)}{g \times h_{fg} \times \rho^2}\right]^{0.25}$$

$$\delta_x = \frac{4 \times 2.33\times10^{-4} \times 0.685 \times 1 \times (127-117)}{9.81 \times 2183\times10^{3} \times (945)^2}$$

$$\delta_x = 1.35\times10^{-4}\ m$$

4. A tube of 2m length and 25 mm outer diameter is to be condense saturated steam at 100°C while the tube surface is maintained at 92°C. Estimate the average heat transfer coefficient and the rate of condensation of steam if tube is kept horizontal. The steam condenses on the outside of the tube.

Given:-

Tube length, $L = 2\,m$

Diameter, $D = 25\,mm = 0.025\,m$

Saturated temp, $T_{sat} = 100°C$

Surface temp., $T_w = 92°C$

To find:

(i) Average H.T. coefficient, h

(ii) Rate of condensation, $\dot{m}$

Solution:

From steam Table page No. 4

Properties of steam at 100°C

$h_{fg} = 2256.9\ KJ/Kg = 2256.9\times10^3\ J/Kg$

W.K.T. Film temperature, $T_f = \frac{T_w + T_{sat}}{2} = \frac{92+100}{2}$

$T_f = 96°C$

properties of saturated water at 96°C

$\rho = 965 \ Kg/m^3$

$\nu = 0.310 \times 10^{-6} \ m^2/s$

$K = 0.677 \ W/mK$

$\mu = \rho \times \nu = 965 \times 0.310 \times 10^{-6} = 2.99 \times 10^{-4} \ NS/m^2$

From HMTDB, P149, For horizontal tubes

$$h = 0.728 \left[\frac{K^3 \rho^2 g \ h_{fg}}{\mu D (T_{sat} - T_w)} \right]^{0.25}$$

$$h = 0.728 \left[\frac{(0.677)^3 \times (965)^2 \times 9.81 \times 2256.9 \times 10^3}{2.99 \times 10^{-4} \times 0.025 \times (100-92)} \right]^{0.25}$$

$h = 13,166.08 \ W/m^2 K$

Heat transfer, $Q = hA(T_{sat} - T_w)$

$= h \times \pi D L (T_{sat} - T_w)$

$= 13,166.08 \times \pi \times 0.025 \times 2 \times (100-92)$

$Q = 16,536.59 \ W$

w.k.t. $Q = \dot{m} \times h_{fg}$

$\therefore \ \dot{m} = \dfrac{Q}{h_{fg}}$

$\dot{m} = \dfrac{16,536.59}{2256.9 \times 10^3}$

Rate of condensation, $\dot{m} = 7.327 \times 10^{-3} \ Kg/s$

5. A vertical plate $0.5 m^2$ in area at temp. of $92°C$ is exposed to steam at atmospheric pressure. If steam is dry and saturated, estimate the heat transfer rate and condensate mass per hour. The vertical length of the plate is 0.5 m. properties of water at film temp. of $96°C$ can be obtained from tables.

Given:

Area, $A = 0.5 m^2$

Film temp, $T_f = 96°C$

Surface temp., $T_w = 92°C$

vertical length, $L = 0.5 m$

To find:

(i) Heat transfer rate

(ii) condensate mass per hour

Solution:

From steam table, properties of saturated water at $96°C$

$\rho = 963.6 \ Kg/m^3$

$\nu = 0.307 \times 10^{-6} m^2/s$

$K = 0.6781 \ W/mK$

$\mu = \rho \times \nu = 2.95 \times 10^{-4} \frac{N \cdot s}{m^2}$ or $Kg/m \cdot s$ or Pascal-second

From steam table, page No. 4

Properties of steam at $100°C$ [$\because T_{sat} = 100°C$]

$h_{fg} = 2256.9 \ KJ/Kg = 2256.9 \times 10^3 J/Kg$

Assume the condensate film is laminar.

From HMTDB, P148

Average H.T. coefficient(h) for vertical surface, laminar flow (Mc adams)

$$h = 1.13 \left[\frac{K^3 \rho^2 g\, h_{fg}}{\mu L (T_{sat} - T_w)}\right]^{0.25}$$

$$h = 1.13 \left[\frac{(0.6781)^3 \times (963.6)^2 \times 9.81 \times 2256.9 \times 10^3}{2.95 \times 10^{-4} \times 0.5 \times (100-92)}\right]^{0.25}$$

$$h = 9694.35 \ W/m^2K$$

(i) Rate of heat transfer, $Q = hA(T_{sat} - T_w)$

$$Q = 9694.35 \times 0.5 \times (100-92)$$

$$Q = 38,770 \text{ Watts}$$

(ii) Condensate mass/hour

$$Q = \dot{m}\, h_{fg}$$

$$\dot{m} = \frac{Q}{h_{fg}} = \frac{38,770}{2256.9 \times 10^3}$$

$$\dot{m} = 0.0171 \ Kg/s$$

$$\dot{m} = 0.0171 \times 3600 \ Kg/hr$$

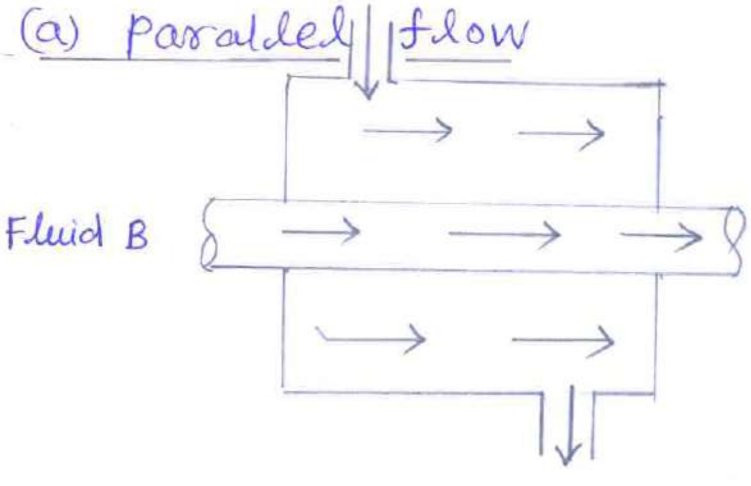
$$\dot{m} = 61.56 \ Kg/hr$$

Nusselt's theory of condensation

1. The plate is maintained at a uniform temperature T_w, which is less than the saturation temp. (T_{sat}) of the vapour.
2. Fluid properties are constant.
3. The shear stress at the liquid vapour interface is negligible.
4. The heat transfer across the condensate layer is by pure conduction and the temp. distribution is linear.
5. The condensing vapour is entirely clean and free from gases, air and non condensing impurities.

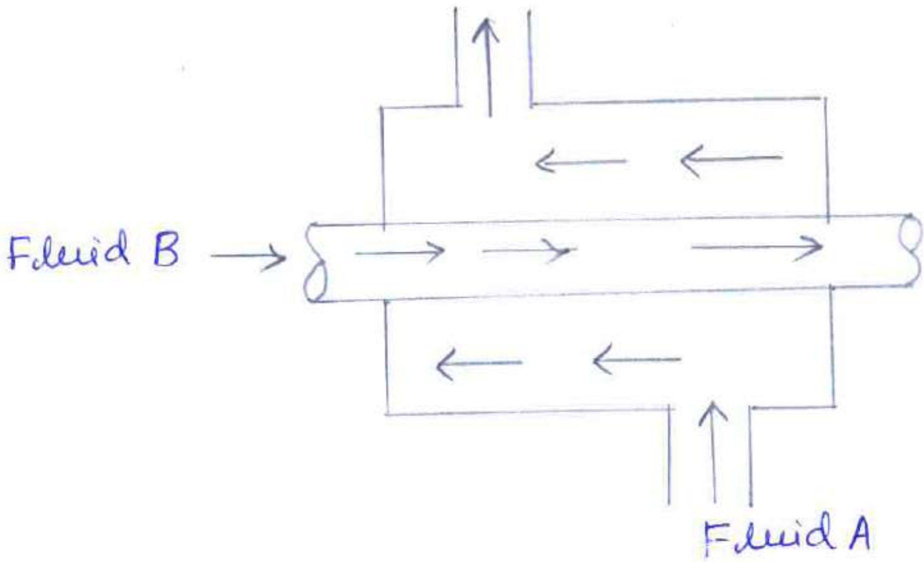
Types of heat exchangers

(a) Parallel flow



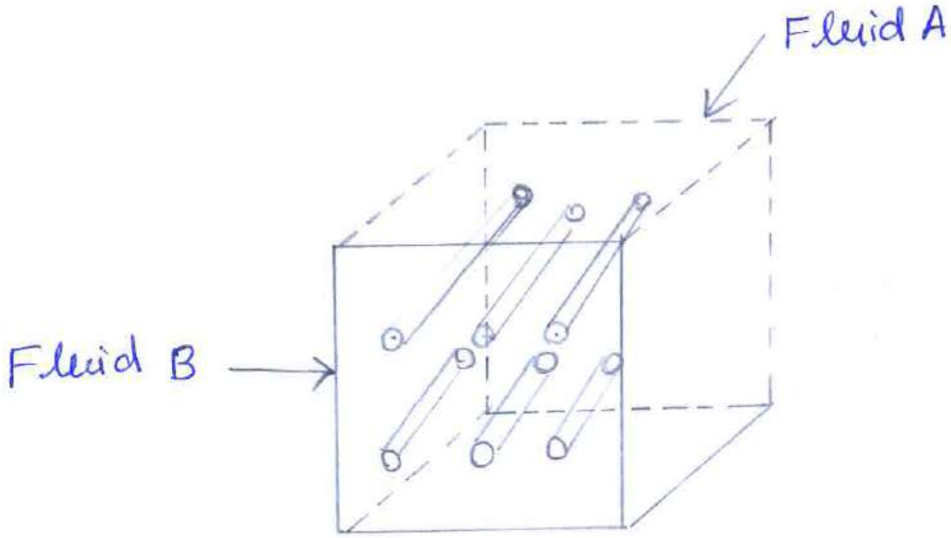
* If both the fluids move in the same direction as shown in figure. This arrangement is called parallel flow type heat exchanger.

(b) counter flow



* In the counterflow arrangement, the fluids move in parallel but opposite direction.

(c) cross flow



* In this arrangement, the fluids move at right angles to each other.
* This cross flow heat exchanger is widely used in air-conditioning applications.

Overall heat transfer coefficient

$$Q = UA\Delta T_m$$

ΔT_m - Average effective temp. difference for the entire heat exchanger

Formula:-

For Plane wall

$$U = \frac{1}{\frac{1}{h_o} + \frac{L}{K} + \frac{1}{h_i}}$$

For cylindrical wall

$$U_o = \frac{1}{\frac{1}{h_o} + \frac{r_o}{K}\ln\left(\frac{r_o}{r_i}\right) + \left(\frac{r_o}{r_i}\right)\frac{1}{h_i}}$$

or

$$U_i = \frac{1}{\frac{1}{h_i} + \frac{r_i}{K}\ln\left(\frac{r_o}{r_i}\right) + \left(\frac{r_i}{r_o}\right)\frac{1}{h_o}}$$

$$U_i A_i = U_o A_o$$

where i - inside surface

o - outside surface

6. Water heated to 80°c flows through a 2.54 cm I.D. and 2.88 cm O.D. steel tube ($K = 50\, W/mK$). The tube is exposed to an environment which is known to provide an average convection coefficient of $h_o = 30800\, W/m^2K$ on the outside of the tube. The water velocity is 50 cm/s.

calculate the overall heat transfer coefficient based on the outer area of the pipe.

Given:

Bulk temp. of water $= 80°C$

Steel tube I.D., $D_i = 2.54\,cm = 0.0254\,m$

$$r_i = \frac{0.0254}{2} = 0.0127\,m$$

Steel tube O.D., $D_o = 2.88\,cm = 0.0288\,m$

$$r_o = \frac{0.0288}{2} = 0.0144\,m$$

Average convection coefficient outside of the tube $\Big\}$ $h_o = 30{,}800\,W/m^2K$

Velocity, $U = 50\,cm/s = 0.5\,m/s$

To find:

Overall H.T. coefficient based on the outer area of the pipe

Solution:

The properties of water at the bulk temp. of $80°C$ are

$\rho = 974\,Kg/m^3$

$K = 668.7 \times 10^{-3}\,W/mK$

$\nu = 0.364 \times 10^{-6}\,m^2/s$

$Pr = 2.20$

Reynold's Number, $Re = \frac{UD}{\nu} = \frac{0.5 \times 0.0254}{0.364 \times 10^{-6}}$

$$Re = 34890 > 2300$$

$\therefore$ The flow is turbulent.

For fully developed flow,

$$Nu_D = 0.023\ Re_D^{4/5}\ Pr^{0.4}$$

$$Nu_D = 0.023 \times (34890)^{0.8} \times (2.2)^{0.4} \qquad \left[\frac{4}{5} = 0.8\right]$$

$$Nu = 135.79$$

Also, $Nu = \frac{hD}{K} \Rightarrow h = \frac{Nu\,K}{D}$

$$h_i = \frac{135.79 \times 668.7 \times 10^{-3}}{0.0254}$$

$$h_i = 3575\ W/m^2K$$

Overall heat transfer coefficient based on the outer area of the pipe, U_o

$$U_o = \frac{1}{\frac{1}{h_o} + \frac{r_o}{K}\ln\left(\frac{r_o}{r_i}\right) + \left(\frac{r_o}{r_i}\right)\frac{1}{h_i}}$$

$$= \frac{1}{\frac{1}{30800} + \frac{0.0144}{50}\ln\left(\frac{0.0144}{0.0127}\right) + \left(\frac{0.0144}{0.0127}\right)\frac{1}{3575}}$$

$$U_o = 2591.9\ W/m^2K$$

Fouling Factors

The surfaces of a heat exchanger do not remain clean after it has been in use for some time. The surfaces become fouled with deposits. The effect of these deposits affecting the value of overall heat transfer coefficient (U). This effect taken care of by introducing an additional thermal resistance called the fouling resistance (R_f).

R_f must be determined experimentally in both clean and dirty conditions.

$$\frac{1}{U_{foul}} = R_f + \frac{1}{U_{clean}}$$

$$U_o = \frac{1}{\frac{1}{h_o} + R_{fo}\ \frac{r_o}{K}\ln\left(\frac{r_o}{r_i}\right) + \left(\frac{r_o}{r_i}\right) R_{fi} + \left(\frac{r_o}{r_i}\right)\frac{1}{h_i}}$$

$$U_i = \frac{1}{\frac{1}{h_i} + R_{fi}\ \frac{r_i}{K}\ln\left(\frac{r_o}{r_i}\right) + \left(\frac{r_i}{r_o}\right) R_{fo} + \left(\frac{r_i}{r_o}\right)\frac{1}{h_i}}$$

7. Determine the overall heat transfer coefficient U_o based on the outer surface of a 2.54 cm O.D. and 2.286 cm I.D. heat exchanger tube ($K = 102$ W/mK), if the heat transfer coefficients at the inside and outside of the tube are $h_i = 5500$ W/m²K and $h_o = 3800$ W/m²K respectively and the fouling factors are $R_{fo} = R_{fi} = 0.0002$ m².W.K.

Given:

Tube O.D., $D_o = 2.54$ cm $= 0.0254$ m

$r_o = 0.0127$ m

Tube I.D., $D_i = 2.286$ cm $= 0.02286$ m

$r_i = 0.01143$ m

Thermal conductivity, $K = 102$ W/mK

Inside H.T. coefficient, $h_i = 5500$ W/m²K

outside H.T. coefficient, $h_o = 3800$ W/m²K

Fouling factor, $R_{fo} = R_{fi} = 0.0002$ m².W.K

To find:

overall H.T. coefficient based on the outside area of the tube,

Solution:-

$$U_o = \frac{1}{\frac{1}{h_o} + R_{fo}\ \frac{r_o}{K} \ln\left(\frac{r_o}{r_i}\right) + \left(\frac{r_o}{r_i}\right) R_{fi} + \left(\frac{r_o}{r_i}\right)\frac{1}{h_i}}$$

$$U_o = \frac{1}{\frac{1}{3800} + 0.0002\left(\frac{0.0127}{102}\right)\ln\left(\frac{0.0127}{0.01143}\right) + \left(\frac{0.0127}{0.01143}\right)0.0002 + \left(\frac{0.0127}{0.01143}\right)\frac{1}{5000}}$$

$$U_o = 1110\ W/m^2K$$

LMTD method:

The temperature difference between the hot and cold fluids in the heat exchanger varies from point to point. In addition various modes of heat transfer are involved. Therefore, based on the concept of appropriate mean temp. difference also called LMTD.

$$Q = UA(\Delta T)_m$$

$(\Delta T)_m$ - Logarithmic mean temp. difference

Parallel and counter flow

Formulae

1. Heat transfer, $Q = UA(\Delta T)_m$

(i) For parallel flow, From HMTDB P152

$$(\Delta T)_{lm} = \frac{(T_1 - t_1) - (T_2 - t_2)}{\ln\left[\frac{T_1 - t_1}{T_2 - t_2}\right]}$$

(ii) counter flow

$$(\Delta T)_{lm} = \frac{(T_1 - t_2) - (T_2 - t_1)}{\ln \left[\frac{T_1 - t_2}{T_2 - t_1}\right]}$$

T_1 – E.T.H.F
T_2 – EX.T.H.F
t_1 – E.T.C.F
t_2 – EX.T.C.F

2. Heat lost by Hot fluid = Heat gained by cold fluid

$$Q_h = Q_c$$

$$m_h C_{ph} (T_1 - T_2) = m_c \cdot C_{pc} (t_2 - t_1)$$

m_h – Mass flow rate of H.F. in Kg/s
m_c – Mass flow rate of C.F. in Kg/s
C_{ph} – Specific heat of H.F. in J/KgK
C_{pc} – Specific heat of C.F. in J/KgK

3. Surface area of the tube

$$A = \pi D_i L$$

where D_i – Inner dia.

4. $Q = \dot{m} \times h_{fg} \Rightarrow \dot{m} = \frac{Q}{h_{fg}}$

5. Mass flow rate, $\dot{m} = \rho A U$

8. A counterflow double pipe heat exchanger using superheated steam is used to heat the water at a rate of 3 Kg/s. The steam enters the exchanger at 150°c and leaves at 130°c. The inlet and exit temperature of water are 30°c and 80°c respectively. The overall heat transfer coefficient is 820 W/m²K. Calculate the heat transfer area required what would be the increase in surface area, if fluids flow in parallel? Take Cp of water 4.187 KJ/KgK

Given:

Mass flow rate of water (cold fluid), $m_c = 3$ Kg/s

Entry temp. of hot steam (Hot), $T_1 = 150°C$

Exit temp. of hot steam (Hot), $T_2 = 130°C$

Entry temp. of water (cold), $t_1 = 30°C$

Exit temp. of water (cold), $t_2 = 80°C$

Overall H.T. coefficient, $U = 820\ W/m^2K$

Specific heat of water (cold), $C_{pc} = 4187\ J/KgK$

To find:

(i) Area for counter flow

(ii) Area for parallel flow

(iii) Increase in surface area

Solution:

(i) For counterflow

$$(\Delta T)_{lm} = \frac{(T_1 - t_2) - (T_2 - t_1)}{\ln\left[\frac{T_1 - t_2}{T_2 - t_1}\right]} = \frac{(150-80) - (130-30)}{\ln\left[\frac{150-80}{130-30}\right]}$$

$$(\Delta T)_{lm} = 84.11°C$$

$$Q = UA(\Delta T)_{lm} \Rightarrow A = \frac{Q}{U(\Delta T)_{lm}} = \frac{6,28,050}{820 \times 84.11}$$

$$A = 9.10\ m^2$$

$$Q = m_c \cdot C_{pc}(t_2 - t_1)$$

$$Q = 3 \times 4187(80-30)$$

$$Q = 6,28,050\ W$$

Area for counterflow, $A = 9.10\ m^2$

(ii) For parallel flow

$$(\Delta T)_{lm} = \frac{(T_1 - t_1) - (T_2 - t_2)}{\ln\left[\frac{T_1 - t_1}{T_2 - t_2}\right]} = \frac{(150-30) - (130-80)}{\ln\left[\frac{(150-30)}{(130-80)}\right]}$$

$$(\Delta T)_{lm} = 79.95°C$$

$$A = \frac{Q}{U(\Delta T)_{lm}} = \frac{6,28,050}{820 \times 79.95} = 9.58\ m^2$$

(iii) Increase in surface area $= \left[\frac{9.58 - 9.1}{9.1}\right] \times 100$

$$= 5.27\%$$

0503 Sachdeva Ex 12.5

9. A counter flow concentric tube heat exchanger is used to cool engine oil ($C = 2130\ J/KgK$) from 160°C to 60°C with water, available at 25°C as the cooling medium. The flow rate of cooling water through the inner tube of 0.5 m diameter is 2 Kg/s while the flow rate of oil through the outer annulus O.D. = 0.7 m is also 2 Kg/s. If the value of the overall heat transfer coefficient is 250 W/m²K, how long must the heat exchanger be to meet its cooling requirement?

Given:

Specific heat of engine oil, $C_{ph} = 2130\ J/Kg.K$

Hot fluid (engine oil) entry temp., $T_1 = 160°C$

Exit temp. of hot fluid, $T_2 = 60°C$

Entry temp. of cooling medium, $t_1 = 25°C$

Mass flow rate of cooling water, $\dot{m}_c = 2\ Kg/s$

Inner tube diameter, $D_i = 0.5\ m$

Mass flow rate of oil (Hot), $\dot{m}_h = 2\ Kg/s$

Overall H.T. coefficient, $U = 250\ W/m^2K$

To find:

Length of the Heat Exchanger

Solution:-

specific heat of water, $C_p = 4186\ J/kg\,K$

Heat lost by oil = Heat gained by cooling water

$$\dot{m}_h C_{ph}(T_1 - T_2) = \dot{m}_c C_{pc}(t_2 - t_1)$$

$$2 \times 2130 \times (160-60) = 2 \times 4186 \times (t_2 - 25)$$

$$t_2 - 25 = \frac{2 \times 2130 \times 100}{2 \times 4186}$$

$$t_2 = 50.88 + 25 = 75.88^\circ C$$

Counter flow H.E.

From HMT DB P152, For counterflow

$$(\Delta T)_{lm} = \frac{(T_1 - t_2) - (T_2 - t_1)}{\ln\left[\frac{T_1 - t_2}{T_2 - t_1}\right]} = \frac{(160-75.88) - (60-25)}{\ln\left[\frac{(160-75.88)}{(60-25)}\right]}$$

$$(\Delta T)_{lm} = 56.01^\circ C, \quad Q = m_c C_{pc}(t_2 - t_1) = 2 \times 4186 \times (75.88 - 25) = 4,25,967.36 \text{ watts}$$

$$Q = UA(\Delta T)_{lm}$$

$$A = \frac{Q}{U(\Delta T)_{lm}} = \frac{4,25,967.36}{250 \times 56.01}$$

$$A = 30.42\ m^2$$

Also, Area, $A = \pi D_i L$

$$\therefore L = \frac{A}{\pi D_i}$$

$$L = \frac{30.42}{\pi \times 0.5}$$

Length of the Heat Exchanger $L = 19.37\ m$

10. Saturated steam at 120°C is condensing on the outer tube surface of a single pass heat exchanger. The heat transfer coefficient is $U_0 = 1800\ W/m^2K$. Determine the surface area of a heat exchanger capable of heating 1000 Kg/hr of water from 20°C to 90°C. Also compute the rate of condensation of steam of $h_{fg} = 2200\ KJ/Kg$.

Given:

Single Pass Heat Exchanger

Saturated steam (Hot), $T_1 = 120°C$

overall H.T. coefficient, $U_0 = 1800\ W/m^2K$

mass flow rate (cold), $\dot{m}_c = 1000\ Kg/hr = \frac{1000}{3600}\ Kg/s$

Entry temp. of cooling medium, $t_1 = 20°C$

Exit temp. of cooling medium, $t_2 = 90°C$

$h_{fg} = 2200\ KJ/Kg = 2200 \times 10^3\ J/Kg$

To find:-

(i) Surface area of Heat Exchanger

(ii) Rate of condensation of steam

Solution: Specific heat of water, $C_p = 4186\ J/KgK$

$$Q = \dot{m}_c \cdot C_{pc}(t_2 - t_1) = \frac{1000}{3600} \times 4186 \times (90-20)$$

$$Q = 81394.4\ \text{watts}$$

$$(\Delta T)_{lm} = \frac{(T_1 - t_1) - (T_1 - t_2)}{\ln\left[\frac{T_1 - t_1}{T_1 - t_2}\right]} = \frac{(120-20)-(120-90)}{\ln\left[\frac{(120-20)}{(120-90)}\right]}$$

$$(\Delta T)_{lm} = \frac{100-30}{\ln(10/3)} = \frac{70}{1.204} = 58.14°C$$

$Q = UA(\Delta T)_{lm}$

$\therefore A = \frac{Q}{U(\Delta T)_{lm}} = \frac{81394.4}{1800 \times 58.14} = 0.78 \, m^2$

(ii) Rate of condensation of steam, $\dot{m}_h$

$Q = \dot{m}_h h_{fg}$

$\dot{m} = \frac{Q}{h_{fg}} = \frac{81394.4}{2200 \times 10^3} = 0.037 \, Kg/s$

$\dot{m} = 133.2 \, Kg/hr$

NTU method:

11. A Parallel flow heat exchanger has hot and cold water stream running through it, the flow rates are 10 and 25 Kg/min respectively. Inlet temperatures are 75°c and 25°c on hot and cold sides. The exit temperature on the hot side should not exceed 50°c. Assume $h_i = h_o = 600 \, W/m^2K$ Calculate the area of heat exchanger using NTU method.

Given:

Mass flow rate of hot water, $\dot{m}_h = 10 \, Kg/min = \frac{10}{60} \, Kg/s$

Mass flow rate of cold water, $\dot{m}_c = 25 \, Kg/min = \frac{25}{60} \, Kg/s$

Entry temp. of hot water, $T_1 = 75°C$

Entry temp. of cold water, $t_1 = 25°C$

Exit temp. of hot water, $T_2 = 50°C$

Inside & outside H.T. coefficient, $h_o = h_i = 600 \, W/m^2K$

To find:

Area of Heat Exchanger using NTU method

Solution:

Specific heat of water, $C_{p_c} = C_{ph} = 4186 \ J/KgK$

Capacity rate of hot fluid, $C = \dot{m}_h \times C_{ph}$

$$C = 0.166 \times 4186$$

$$C = 694.87 \ W/K \quad \text{—} \ ①$$

Capacity rate of cold fluid, $C = \dot{m}_c \times C_{pc}$

$$C = 0.46 \times 4186$$

$$C = 1741.37 \ W/K \quad \text{—} \ ②$$

From eqn ① and ②, $C_{min} = 694.87 \ W/K$, $C_{max} = 1741.37 \ W/K$

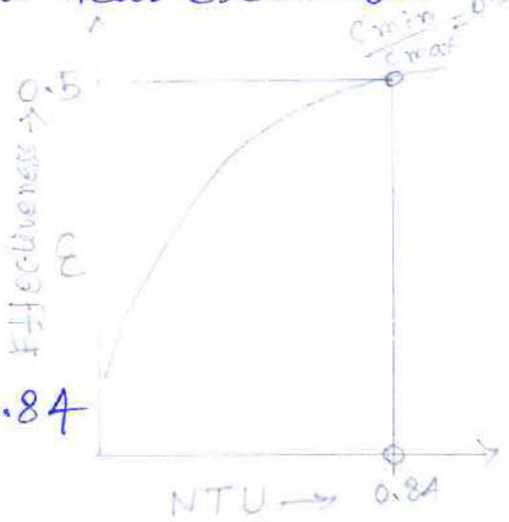
$$\frac{C_{min}}{C_{max}} = \frac{694.87}{1741.37} = 0.399$$

From HMT DB, P152

$$\varepsilon = \frac{T_1 - T_2}{T_1 - t_1} = \frac{75-50}{75-25} = \frac{25}{50} = 0.5$$

From HMT DB P162, parallel flow heat exchanger

From graph,



y axis $\rightarrow \varepsilon = 0.5$

Curve $\rightarrow \frac{C_{min}}{C_{max}} = 0.399$

Corresponding NTU (x-axis) $= 0.84$

From HMT DB P151

NTU (Number of Transfer units) $= \frac{UA}{C_{min}}$ — ①

Overall heat transfer coefficient

$$\frac{1}{U} = \frac{1}{h_i} + \frac{1}{h_o}$$

$$\frac{1}{U} = \frac{h_o + h_i}{h_i h_o}$$

$$\therefore U = \frac{h_i h_o}{h_o + h_i}$$

$$U = \frac{600 \times 600}{600 + 600}$$

$$U = 300 \ W/m^2K$$

Substitute NTU, C_{min} and U values in equation ①

$$NTU = \frac{UA}{C_{min}}$$

$$\therefore A = \frac{NTU \times C_{min}}{U}$$

$$A = \frac{0.84 \times 694.87}{300}$$

Heat exchanger Area, $A = 1.945 \ m^2$

UNIT - IV RADIATION

Definition:-

* Heat transfer from one body to another without any transmitting medium is known as radiation. It is an electro-magnetic wave phenomenon.

Emission properties:-

* The rate of emission of radiation by a body depends upon the following factors.

1. wave length or frequency of radiation
2. The temperature of surface
3. The nature of the surface

Black body radiation:-

Concept of blackbody

1. A black body absorbs all incident radiation, regardless of wave length and direction.
2. For a prescribed temperature and wave length, no surface can emit more energy than black body.

* A black body is regarded as a perfect absorber of incident radiation
* A black body is a perfect emitter.

Stefan boltzmann law

* The emissive power of a black body is proportional to the fourth power of absolute temperature.

$E_b \propto T^4$ $\quad \therefore E_b = \sigma T^4$

E_b - Emissive power in W/m^2

σ - Stefan boltzmann constant

$\sigma = 5.67 \times 10^{-8}\ W/m^2 K^4$

T - Temperature in K

Emissivity:

* It is defined as the ability of the surface of a body to radiate heat.

Grey body radiation:-

* If a body absorbs a definite percentage of incident radiation irrespective of their wave length, the body is Known as grey body.
* The emissive power of a grey body is always less than that of the black body.

Formulae:-

1. Emissive power or Total Emissive Power

$$E_b = \sigma T^4$$

2. Wien's law

$$\lambda_{max} \cdot T = 2898\,\mu m K = 2.9 \times 10^{-3}\, mK$$

3. Monochromatic emissive power (or) spectral emissive power

$$E_{b\lambda} = \frac{C_1 \lambda^{-5}}{e^{\left[\frac{C_2}{\lambda T}\right]} - 1}$$

$$C_1 = 0.374 \times 10^{-15}\, W m^2$$

$$C_2 = 14.4 \times 10^{-3}\, mK$$

4. Maximum Emissive power

$$E_{b\lambda} = C_4 T^5$$

where $C_4 = 1.307 \times 10^{-5}$

5. Intensity of radiation

$$I_n = \frac{E_b}{\pi}$$

6. Absorptivity, $\alpha = \dfrac{\text{Radiation absorbed}}{\text{Incident radiation}}$

Reflectivity, $\rho = \dfrac{\text{Radiation reflected}}{\text{Incident radiation}}$

Transmissivity, $\tau = \dfrac{\text{Radiation transmitted}}{\text{Incident radiation}}$

Problem:

1. A black body at 3000 K emits radiation. Calculate the following:

(i) Mono chromatic emissive Power at 1 μm wave length

(ii) wave length at which emission is maximum

(iii) Max. emissive power

(iv) Total emissive power

Given:-

surface temp., $T = 3000\,K$

To find:-

(i) $E_{b\lambda}$

(ii) λ_{max}

(iii) $(E_{b\lambda})_{max}$

(iv) E_b

Solution:

(i) $E_{b\lambda}$

From HMT DB P83

Plank's distribution law,

$$E_{b\lambda} = \frac{c_1 \lambda^{-5}}{e^{\left[\frac{c_2}{\lambda T}\right]} - 1} \qquad \lambda = 1\,\mu m = 1 \times 10^{-6}\,m$$

$$E_{b\lambda} = \frac{0.374 \times 10^{-15} \times (1 \times 10^{-6})^{-5}}{e^{\left[\frac{14.4 \times 10^{-3}}{1 \times 10^{-6} \times 3000}\right]} - 1} = 3.10 \times 10^{12}\,W/m^2$$

(ii) λ_{max}

From wien's law

$$\lambda_{max} \cdot T = 2.9 \times 10^{-3}$$

$$\therefore \lambda_{max} = \frac{2.9 \times 10^{-3}}{3000} = 0.966 \times 10^{-6} \text{ m}$$

(iii) $(E_{b\lambda})_{max}$

$$(E_{b\lambda})_{max} = 1.307 \times 10^{-5} T^5$$

$$= 1.307 \times 10^{-5} \times (3000)^5$$

$$= 3.17 \times 10^{12} \text{ W/m}^2$$

Total Emissive Power (E_b)

From stefen boltzmann law

$$E_b = \sigma T^4$$

$$= 5.67 \times 10^{-8} \times (3000)^4$$

$$= 4.59 \times 10^6 \text{ W/m}^2$$

2. A furnace wall emits radiation at 2000 K. Treating it as black body radiation, calculate

(i) Monochromatic radiant flux density at 1 μm wave length

(ii) wave length at which emission is maximum and the corresponding emissive power

(iii) Total emissive power

Given:

$$T = 2000 \text{ K}$$

$$\lambda = 1 \mu\text{m} = 1 \times 10^{-6} \text{ m}$$

Solution:-

(i) $E_{b\lambda}$

From HMTDB P83

Planck's distribution law

w.k.t. $$E_{b\lambda} = \frac{C_1 \lambda^{-5}}{\left[e^{\left(\frac{C_2}{\lambda T}\right)} - 1\right]}$$

$$E_{b\lambda} = \frac{0.374 \times 10^{-15} \times [1 \times 10^{-6}]^{-5}}{\left[e^{\left(\frac{14.4 \times 10^{-3}}{1 \times 10^{-6} \times 2000}\right)} - 1\right]}$$

$$E_{b\lambda} = 2.79 \times 10^{11}\ w/m^2$$

(ii) λ_{max}

From wien's law,

$$\lambda_{max} \cdot T = 2.9 \times 10^{-3}$$

$$\lambda_{max} = \frac{2.9 \times 10^{-3}}{2000} = 1.45\ \mu m = 1.45 \times 10^{-6}\ m$$

Corresponding emissive power

$$E_{b\lambda} = \frac{C_1 \lambda_{max}^{-5}}{e^{\left(\frac{C_2}{\lambda_{max} \cdot T}\right)} - 1}$$

$$E_{b\lambda} = \frac{0.374 \times 10^{-15} \times (1.45 \times 10^{-6})^{-5}}{e^{\left(\frac{14.4 \times 10^{-3}}{1.45 \times 10^{-6} \times 2000}\right)} - 1}$$

$$E_{b\lambda} = 4.09 \times 10^{11}\ w/m^2$$

(iii) E_b (Total Emissive Power)

From Stefen boltzmann law,

$E_b = \sigma T^4$

$E_b = 5.67 \times 10^{-8} \times (2000)^4$

$E_b = 907.2 \times 10^3 \ W/m^2$

3. The temperature of a black surface $0.25 \ m^2$ of area is $650°c$. calculate,

(i) The total rate of energy emission

(ii) The intensity of normal radiation

(iii) The wave length of maximum monochromatic emissive power

Given:

Area, $A = 0.25 \ m^2$

Temp., $T = 650°c + 273 = 923 \ K$

To find:

(i) Rate of energy emission (E_b)

(ii) Intensity of radiation, I_n

(iii) Max. wave length, λ_{max}

Solution:

Total Emissive Power, $E_b = \sigma T^4$

$= 5.67 \times 10^{-8} \times (923)^4$

$E_b = 41151.89 \ W/m^2$

Total rate of energy emission for $0.25 \ m^2$ area

$E_b = 41151.89 \times 0.25$

$E_b = 10.28 \times 10^3$ watts

(ii) Intensity of radiation (I_n) $\quad I_n = \frac{\text{Rate of energy emiss}}{\pi}$

$$I_n = \frac{E_b}{\pi} = \frac{10.28 \times 10^3}{\pi} = 3274.76 \text{ watts}$$

(iii) Max. wave length (λ_{max})

From wien's law

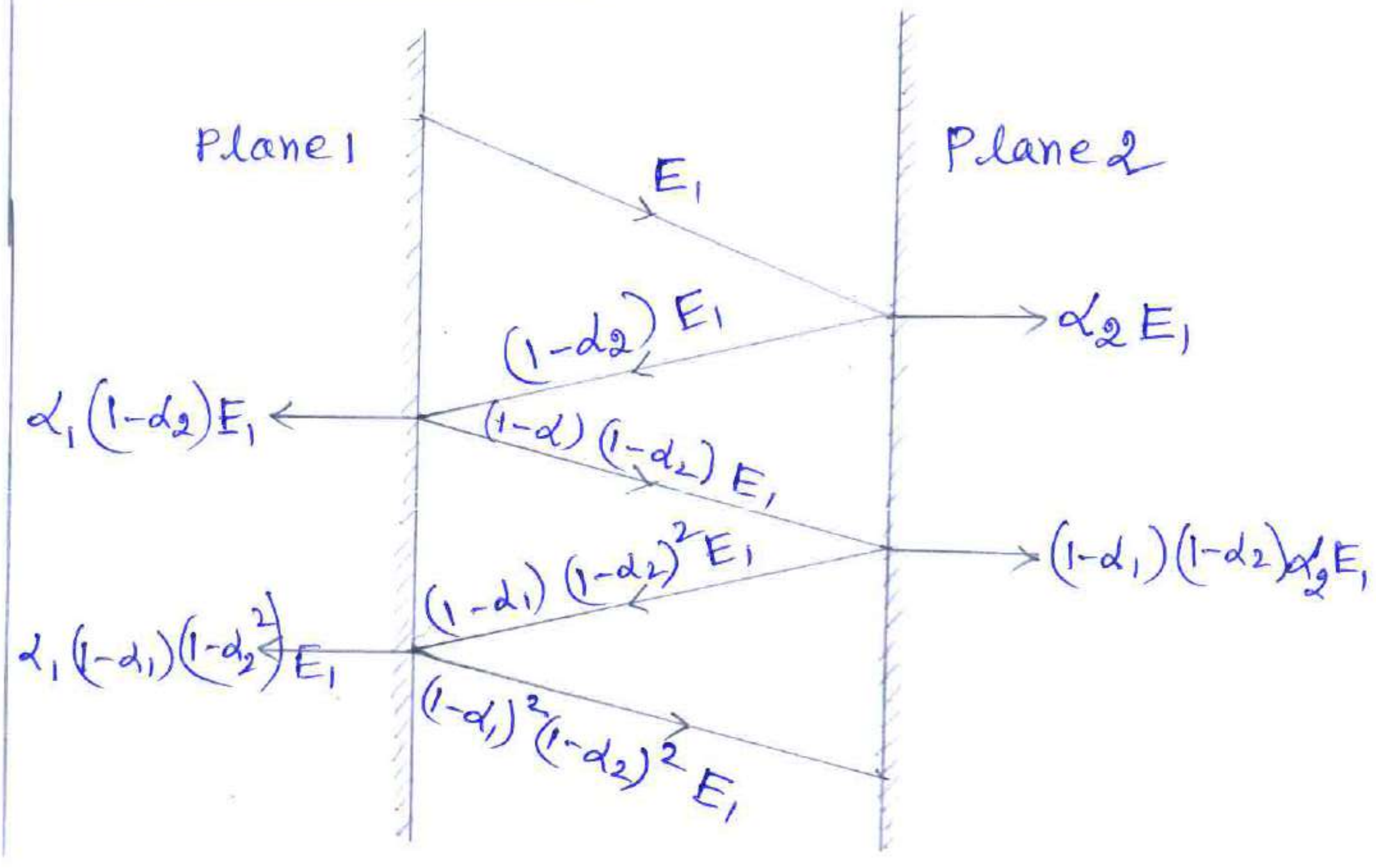
$$\lambda_{max} \cdot T = 2.9 \times 10^{-3} \text{ mK}$$

$$\lambda_{max} = \frac{2.9 \times 10^{-3}}{923} = 3.13 \times 10^{-6} \text{ m}$$

Shape factor:

* The fraction of radiative energy that is diffused from one surface and strikes the other surface directly with no intervening reflections.

Heat exchange between two non-black parallel planes



Let T_1, α_1 and E_1 be the temp, absorptivity and emissivity of the surface 1

Similarly T_2, α_2 and E_2 belongs to surface 2

Assumptions:-

1. The configuration factor of either surface is unity
2. There is no absorbing medium in b/w the surfaces
3. The emissive and reflective properties are constant for overall surfaces.

* The surface 1 emits radiant energy E_1 which falls on the surface 2. out of this, a part of $\alpha_2 E_1$ is absorbed by the surface 2 and the remainder $(1-\alpha_2)E_1$ is reflected back to surface 1.
* on reaching surface 1, a part $\alpha_1(1-\alpha_2)E_1$ is absorbed and the remainder $(1-\alpha_1)(1-\alpha_2)E_1$ is reflected. This process will go on continuing

The rate of radiant energy leaving surface 1 is given by

$$Q_1 = E_1 - \left[\alpha(1-\alpha_2)E_1 + \alpha_1(1-\alpha_1)(1-\alpha_2)^2 E_1 + \alpha_1(1-\alpha_1)^2(1-\alpha_2)^3 E_1 + \cdots\cdots\right]$$

$$= E_1 - \alpha_1(1-\alpha_2)E_1\left[1 + (1-\alpha_1)(1-\alpha_2) + (1-\alpha_1)^2(1-\alpha_2)^2 + \cdots\cdots\right]$$

$$Q_1 = E_1 - \alpha_1 (1-\alpha_2) E_1 [1+p+p^2+\dots\dots] \quad \text{— ①}$$

where $\underline{p = (1-\alpha_1)(1-\alpha_2)}$

$\because$ α_1 and α_2 are less than unity

$p < 1$, the series $1+p+p^2$

when extended to infinity gives

$$\frac{1}{1-p}$$

$$① \Rightarrow Q_1 = E_1 - \alpha_1 (1-\alpha_2) E_1 \times \frac{1}{1-p}$$

$$= E_1 - \frac{\alpha_1 (1-\alpha_2) E_1}{1-p}$$

Substitute 'p' value

$$Q_1 = E_1 \left[1 - \frac{\alpha_1(1-\alpha_2)}{1-(1-\alpha_1)(1-\alpha_2)}\right]$$

From Kirchoff's law, W.K.T.

$$\alpha_1 = \varepsilon_1 \text{ and } \alpha_2 = \varepsilon_2$$

$$Q_1 = E_1 \left[1 - \frac{\varepsilon_1 (1-\varepsilon_2)}{1-(1-\varepsilon_1)(1-\varepsilon_2)}\right]$$

$$Q_1 = E_1 \left[\frac{1-(1-\varepsilon_1)(1-\varepsilon_2) - \varepsilon_1(1-\varepsilon_2)}{1-(1-\varepsilon_1)(1-\varepsilon_2)}\right]$$

$$Q_1 = E_1 \left[\frac{1-[1-\varepsilon_2-\varepsilon_1+\varepsilon_1\varepsilon_2]-\varepsilon_1(1-\varepsilon_2)}{1-[1-\varepsilon_2-\varepsilon_1+\varepsilon_1\varepsilon_2]} \right]$$

$$Q_1 = E_1 \left[\frac{1-1+\varepsilon_2+\varepsilon_1-\varepsilon_1\varepsilon_2-\varepsilon_1+\varepsilon_1\varepsilon_2}{1-1+\varepsilon_2+\varepsilon_1-\varepsilon_1\varepsilon_2} \right]$$

$$Q_1 = \frac{E_1 \varepsilon_2}{\varepsilon_1+\varepsilon_2-\varepsilon_1\varepsilon_2} \quad \text{---} \quad (2)$$

Similarly

$$Q_2 = \frac{E_2 \varepsilon_1}{\varepsilon_1+\varepsilon_2-\varepsilon_1\varepsilon_2} \quad \text{---} \quad (3)$$

The net radiative heat exchange from surface 1 to 2 is given by

$$Q_{12} = Q_1 - Q_2$$

$$= \frac{E_1 \varepsilon_2 - E_2 \varepsilon_1}{\varepsilon_1+\varepsilon_2-\varepsilon_1\varepsilon_2} \quad \text{---} \quad (4)$$

From stefan - boltzmann law,

$$E_b = \sigma T^4$$

$$E_1 = \varepsilon_1 \sigma T_1^4$$

$$E_2 = \varepsilon_2 \cdot \sigma T_2^4$$

Substitute E_1 & E_2 values in eqn (4)

$$Q_{1-2} = \frac{\varepsilon_1 \sigma T_1^4 \varepsilon_2 - \varepsilon_2 \sigma T_2^4 \varepsilon_1}{\varepsilon_1+\varepsilon_2-\varepsilon_1\varepsilon_2}$$

$$= \frac{\varepsilon_1 \varepsilon_2 \sigma (T_1^4 - T_2^4)}{\varepsilon_1+\varepsilon_2-\varepsilon_1\varepsilon_2}$$

$$Q_{12} = \frac{\varepsilon_1 \varepsilon_2}{\varepsilon_1 + \varepsilon_2 - \varepsilon_1 \varepsilon_2} \times \sigma (T_1^4 - T_2^4)$$

$$Q_{12} = \bar{\varepsilon} \sigma (T_1^4 - T_2^4) \quad \text{--- (5)}$$

where $\bar{\varepsilon} = \frac{\varepsilon_1 \varepsilon_2}{\varepsilon_1 + \varepsilon_2 - \varepsilon_1 \varepsilon_2}$

Divide by $\varepsilon_1 \varepsilon_2$

$$\bar{\varepsilon} = \frac{1}{\frac{\varepsilon_1 + \varepsilon_2 - \varepsilon_1 \varepsilon_2}{\varepsilon_1 \varepsilon_2}}$$

$$\bar{\varepsilon} = \frac{1}{\frac{1}{\varepsilon_2} + \frac{1}{\varepsilon_1} - 1}$$

If considering Area, From eqn (5)

$$\boxed{Q_{12} = \bar{\varepsilon} \sigma A (T_1^4 - T_2^4) \quad \text{--- (6)}}$$

Electrical network analogy

* An alternate approach for analysing thermal radiation between gray or black surfaces is called electrical network analogy.

Irradiation (G)

* It is defined as the total radiation incident upon a surface per unit time per unit area.

Radiosity (J)

* It is defined as the total radiation leaving a surface per unit time per unit area

It consists of two parts.

1. Reflected by the surface $= \rho \cdot G$
2. Emitted " " " $= \varepsilon E_b$

$$J = \rho G + \varepsilon E_b \quad \text{--- (1)}$$

W.K.T. $\alpha + \rho + \tau = 1$

$$\alpha + \rho = 1 \qquad (\because \tau = 0)$$

$$\rho = 1 - \alpha$$

$$(1) \Rightarrow J = (1-\alpha) G + \varepsilon E_b$$

W.K.T. $\alpha = \varepsilon$

$$J = (1-\varepsilon) G + \varepsilon E_b$$

Radiosity, $J = \varepsilon E_b + (1-\varepsilon) G \quad \text{--- (2)}$

$$J - \varepsilon E_b = (1-\varepsilon) G$$

Irradiation, $G = \dfrac{J - \varepsilon E_b}{1-\varepsilon} \quad \text{--- (3)}$

The net energy leaving a surface is,

$$\frac{Q_{12}}{A} = J - G$$

$$= J - \left[\frac{J - \varepsilon E_b}{1-\varepsilon}\right]$$

$$\frac{Q_{12}}{A} = \frac{J(1-\varepsilon)-(J-\varepsilon E_b)}{1-\varepsilon}$$

$$= \frac{\cancel{J}-J\varepsilon-\cancel{J}+\varepsilon E_b}{1-\varepsilon}$$

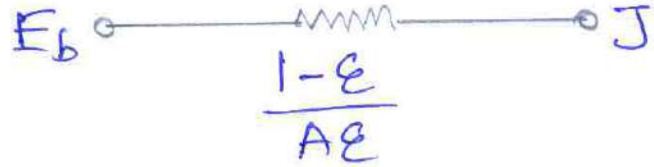
$$= \frac{\varepsilon E_b - J\varepsilon}{1-\varepsilon}$$

$$\frac{Q_{12}}{A} = \frac{\varepsilon(E_b-J)}{1-\varepsilon}$$

$$Q_{12} = \frac{A\varepsilon(E_b-J)}{1-\varepsilon}$$

$$Q_{12} = \frac{E_b-J}{\frac{1-\varepsilon}{A\varepsilon}} \quad \text{--- (4)}$$

This is represented in the form of electrical circuit as shown in fig.



where $\frac{1-\varepsilon}{A\varepsilon}$ = surface resistance of the body

Shape factor:

* If two bodies which are radiating heat with each other and if the radiating heat of one body per unit area is not falling on the other and part of it has gone elsewhere, then it is taken into account by a factor which is known as shape factor.

Heat radiated by the first body and received by the second body $\Big\} = J_1 A_1 F_{1-2}$

Heat radiated from second body and received by first body $\Big\} = J_2 A_2 F_{2-1}$

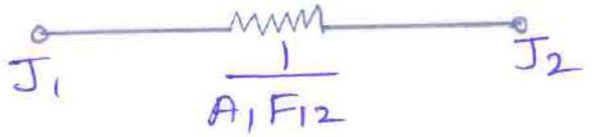
So, net heat lost by the first body

$$Q_{1-2} = J_1 A_1 F_{1-2} - J_2 A_2 F_{2-1}$$

$$= A_1 F_{1-2}(J_1 - J_2) \qquad [\because A_1 F_{12} = A_2 F_{21}]$$

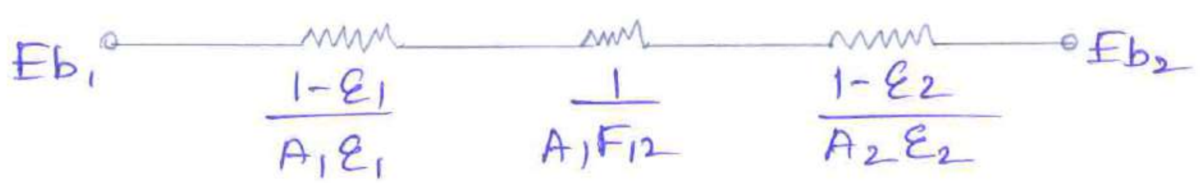
$$Q_{12} = \frac{J_1 - J_2}{\frac{1}{A_1 F_{12}}} \qquad \text{——} (5)$$

This is represented by



where $\frac{1}{A_1 F_{12}}$ - space resistance

If two surface resistance of two bodies and one space resistance between them is considered, the net heat flow is



$$Q_{1-2} = \frac{E_{b1} - E_{b2}}{\frac{1-\varepsilon_1}{\varepsilon_1 A_1} + \frac{1}{A_1 F_{12}} + \frac{1-\varepsilon_2}{\varepsilon_2 A_2}} \qquad [\because E_b = \sigma T^4]$$

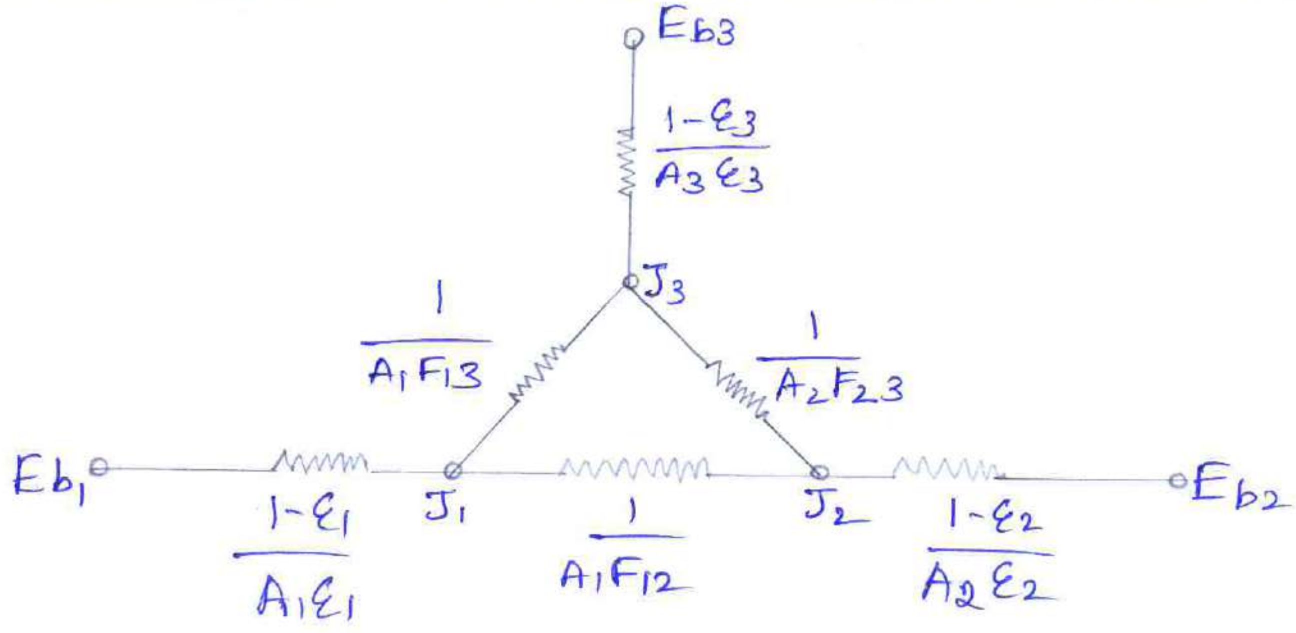
$$Q_{1-2} = \frac{\sigma T_1^4 - \sigma T_2^4}{\frac{1-\varepsilon_1}{\varepsilon_1 A_1} + \frac{1}{A_1 F_{12}} + \frac{1-\varepsilon_2}{\varepsilon_2 A_2}} \quad \text{--- (6)}$$

For black surface, $\varepsilon_1 = \varepsilon_2 = 1$

$$\therefore Q_{12} = \frac{\sigma T_1^4 - \sigma T_2^4}{0 + \frac{1}{A_1 F_{12}} + 0}$$

$$Q_{12} = \sigma (T_1^4 - T_2^4) \times A_1 F_{12}$$

Radiation of heat exchange for three Grey Surfaces



The network for 3 grey surfaces is shown in figure.

$$Q_{12} = \frac{J_1 - J_2}{\frac{1}{A_1 F_{12}}}$$

$$Q_{13} = \frac{J_1 - J_3}{\frac{1}{A_1 F_{13}}}$$

$$Q_{23} = \frac{J_2 - J_3}{\frac{1}{A_2 F_{23}}}$$

The values of Q_{12}, Q_{13}, Q_{23} are determined from the values of the radiosities (J_1, J_2 & J_3)

4. Two parallel plates of size 1 m by 1 m spaced 0.5 m apart are located in a very large room, the walls of which are maintained at a temp. of 27°C. One plate is maintained at a temperature of 900°C and the other at 400°C. Their emissivities are 0.2 and 0.5 respectively. If the plates exchange heat between themselves and surroundings. Find the net heat transfer to each plate and to the room. Consider only the plate surfaces facing each other.

Solution:

Size of the plates = 1 m × 1 m

Distance between plates = 0.5 m

Room temperature, $T_3 = 27°C + 273 = 300\,K$

First plate temp., $T_1 = 900°C + 273 = 1173\,K$

Second plate temp., $T_2 = 400°C + 273 = 673\,K$

First plate Emissivity, $\varepsilon_1 = 0.2$

second plate Emissivity, $\varepsilon_2 = 0.5$

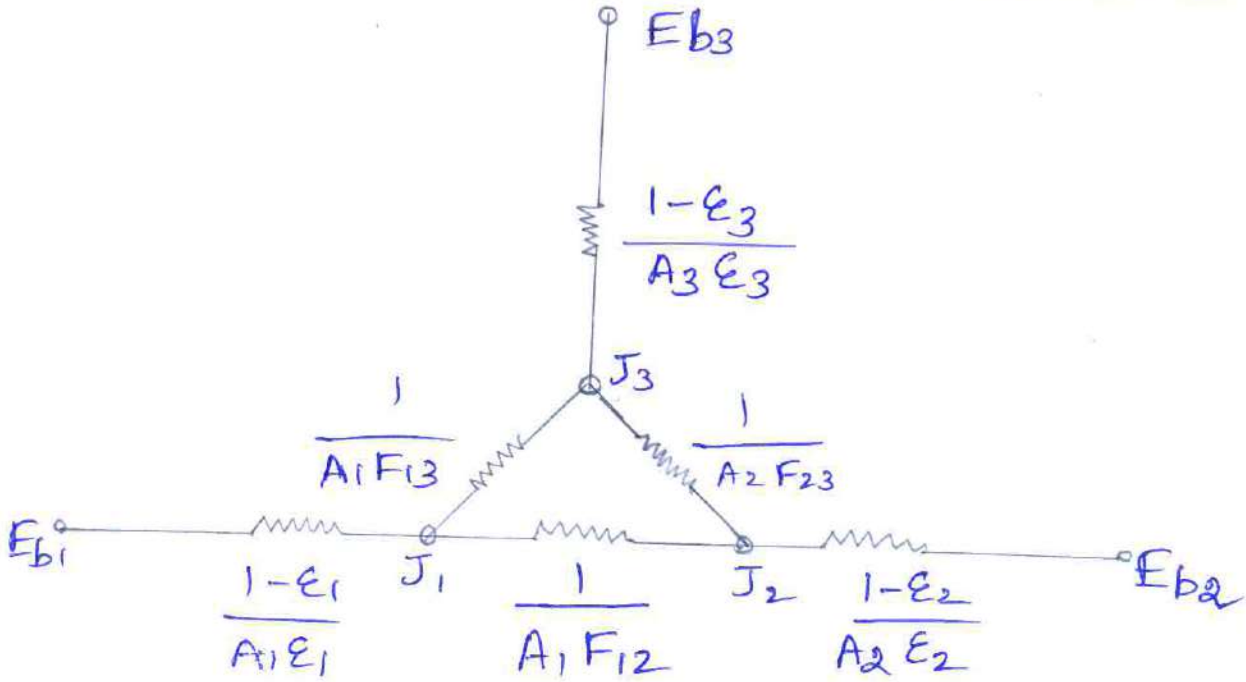
To find:

(i) Net heat transfer to each plate

(ii) Net heat transfer to room

Solution:

Heat exchange takesplace between 2 plates and the room. The radiation network is



Electrical network diagram

Area, $A_1 = 1 \times 1 = 1\,m^2$

$A_1 = A_2 = 1\,m^2$

The room is very large

$\therefore\ A_3 = \infty$

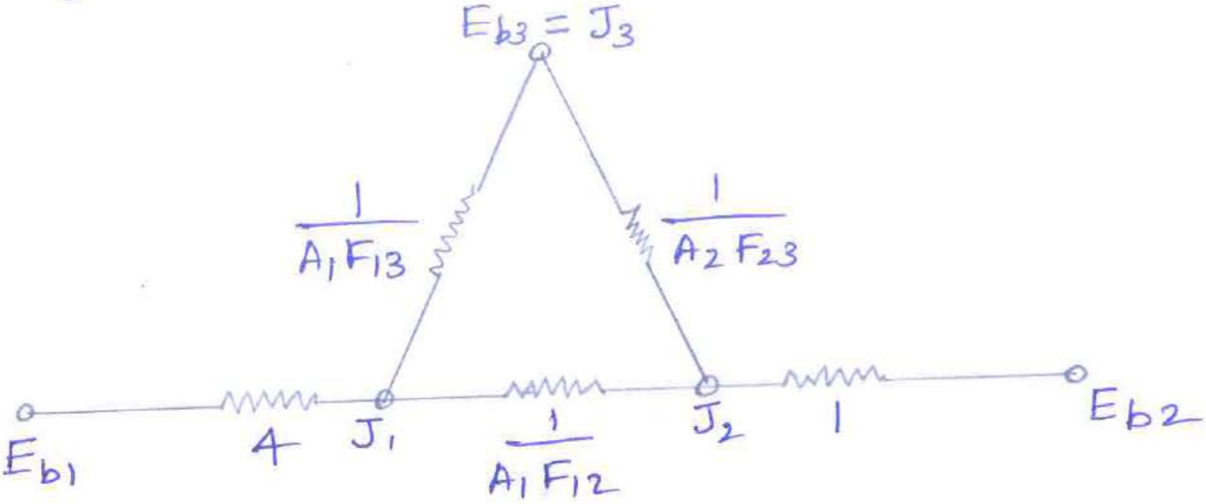
From network diagram

$$\frac{1-\varepsilon_1}{A_1\varepsilon_1} = \frac{1-0.2}{1\times 0.2} = 4$$

$$\frac{1-\varepsilon_2}{A_2\varepsilon_2} = \frac{1-0.5}{1\times 0.5} = 1$$

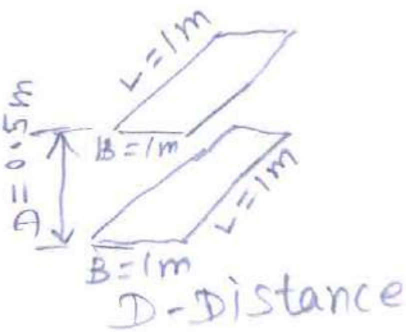
$$\frac{1-\varepsilon_3}{A_3\varepsilon_3} = 0 \quad [\because A_3 = \infty]$$

Apply the values in network diagram



From HMTDB, P 92&93

To find shape factor F_{12}



$$X = \frac{L}{D} = \frac{1}{0.5} = 2$$

$$Y = \frac{B}{D} = \frac{1}{0.5} = 2$$

corresponding shape factor value for the above x and y value is

$$F_{12} = 0.41525$$

W.K.T. $F_{11} + F_{12} + F_{13} = 1$

$F_{11} = 0$; $F_{13} = 1 - F_{12}$; $F_{13} = 1 - 0.41525$

$$F_{13} = 0.5847$$

Similarly, $F_{21} + F_{22} + F_{23} = 1$

$F_{21} = F_{12}$ $\qquad$ $F_{23} = 1 - F_{21}$

$F_{21} = 0.41525$ $\qquad$ $F_{23} = 1 - F_{12}$

$$F_{23} = 1 - 0.41525$$

$$F_{23} = 0.5847$$

From Electrical network diagram,

$$\frac{1}{A_1F_{13}} = \frac{1}{1\times 0.5847} = 1.7102$$

$$\frac{1}{A_2F_{23}} = \frac{1}{1\times 0.5847} = 1.7102$$

$$\frac{1}{A_1F_{12}} = \frac{1}{1\times 0.41525} = 2.408$$

From stefen boltzmann law,

$$E_{b1} = \sigma T_1^4 = 5.67\times10^{-8}\times(1173)^4 = 107.34\times10^3 \, W/m$$

$$E_{b2} = \sigma T_2^4 = 5.67\times10^{-8}\times(673)^4 = 11.63\times10^3 \, W/m^2$$

$$E_{b3} = \sigma T_3^4 = 5.67\times10^{-8}\times(300)^4 = 459.27 \, W/m^2$$

$$E_{b3} = J_3 = 459.27 \, W/m^2$$

The radiosities J_1 and J_2 can be calculated by Kirchoff's law,

At node J_1:

$$\frac{E_{b1}-J_1}{4} + \frac{J_2-J_1}{\frac{1}{A_1F_{12}}} + \frac{E_{b3}-J_1}{\frac{1}{A_1F_{13}}} = 0$$

$$-1.2497 J_1 + 0.415 J_2 = -27.1\times10^3 \quad \text{—} (1)$$

At node J_2:

$$\frac{J_1-J_2}{\frac{1}{A_1F_{12}}} + \frac{E_{b3}-J_2}{\frac{1}{A_2F_{23}}} + \frac{E_{b2}-J_2}{2} = 0$$

$$0.415 J_1 - 1.4997 J_2 = -6.08\times10^3 \quad \text{—} (2)$$

Solving the equations (1) and (2), we get

$$-1.249 J_1 + 0.415 J_2 = -27.10 \times 10^3$$

$$0.415 J_1 - 1.4997 J_2 = -6.08 \times 10^3$$

(1) $\times$ 0.415 $\Rightarrow$

(2) $\times$ 1.249 $\Rightarrow$

$$J_2 = 11.06 \times 10^3 \ W/m^2$$

$$J_1 = 25.35 \times 10^3 \ W/m^2$$

Heat lost by Plate 1

$$Q_1 = \frac{E_{b1} - J_1}{\frac{1-\varepsilon_1}{A_1 \varepsilon_1}} = \frac{107.34 \times 10^3 - 25.35 \times 10^3}{\frac{1-0.2}{1 \times 0.2}}$$

$$Q_1 = 20.49 \times 10^3 \ W$$

Heat lost by Plate 2

$$Q_2 = \frac{E_{b2} - J_2}{\frac{1-\varepsilon_2}{A_2 \varepsilon_2}} = \frac{11.63 \times 10^3 - 11.06 \times 10^3}{\frac{1-0.5}{1 \times 0.5}}$$

$$Q_2 = 570 \ W$$

Total heat lost, $Q = Q_1 + Q_2 = 20.49 \times 10^3 + 570$

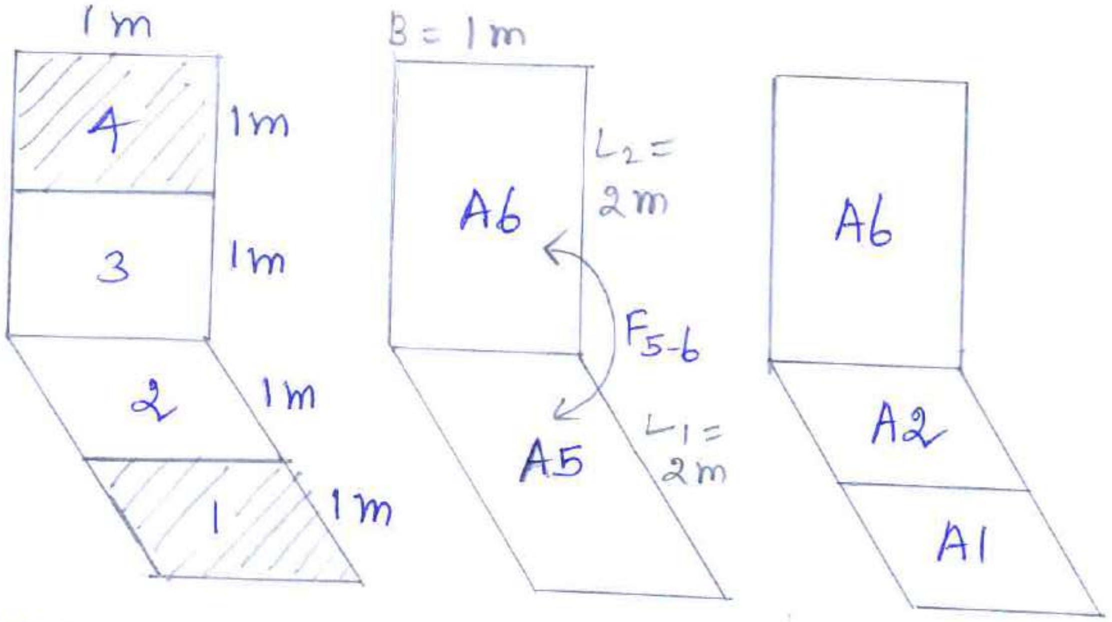
$$Q = 21.06 \times 10^3 \ W$$

Total heat received by the room $\Big\}$ $Q = \dfrac{J_1 - J_3}{\frac{1}{A_1 F_{13}}} + \dfrac{J_2 - J_3}{\frac{1}{A_2 F_{23}}}$

$$Q = \frac{25.35 \times 10^3 - 459.27}{1.71} + \frac{11.06 \times 10^3 - 459.27}{1.71} \quad [\because E_{b3} = J_3]$$

$$Q = 20.75 \times 10^3 \ W$$

5. Find the shape factor F_{14} for the figure shown below.



Solution:

From figure, $A_5 = A_1 + A_2$; $A_6 = A_3 + A_4$

$A_5 F_{56} = A_1 F_{1-6} + A_2 F_{2-6}$ $\quad [\because F_{5-6} = F_{1-6} + F_{2-6}]$

$= A_1 F_{1-3} + A_1 F_{1-4} + A_2 F_{2-6}$ $\quad [\because F_{16} = F_{1-3} + F_{1-4}]$

$A_5 F_{5-6} = A_5 F_{5-3} - A_2 F_{2-3} + A_1 F_{1-4} + A_2 F_{2-6}$

$[\because A_1 = A_5 - A_2;\ F_{1-3} = F_{5-3} - F_{2-3}]$

$A_1 F_{1-4} = A_5 F_{5-6} - A_5 F_{5-3} + A_2 F_{2-3} - A_2 F_{2-6}$

$$F_{1-4} = \frac{A_5}{A_1}\left[F_{5-6} - F_{5-3}\right] + \frac{A_2}{A_1}\left[F_{2-3} - F_{2-6}\right] \quad \text{--- (1)}$$

From HMT DB P96, Shape factor (F_{5-6}) for A5 & A6

$Z = \frac{L_2}{B} = \frac{2}{1} = 2$, $Y = \frac{L_1}{B} = \frac{2}{1} = 2$

Corresponding shape factor value is

$F_{5-6} = 0.14930$

Shape factor (F_{5-3}) for A5 and A3

$Z = \frac{L_2}{B} = \frac{1}{1} = 1$, $Y = \frac{L_1}{B} = \frac{2}{1} = 2$

$F_{5-3} = 0.11643$

Shape factor (F_{2-3}) for A_2 and A3

$Z = \frac{L_2}{B} = \frac{1}{1} = 1$, $Y = \frac{L_1}{B} = \frac{1}{1} = 1$

$F_{2-3} = 0.20004$

Shape factor (F_{2-6}) for A_2 and A_6

$Z = \frac{L_2}{B} = \frac{2}{1} = 2$, $Y = \frac{L_1}{B_1} = \frac{1}{1} = 1$

$F_{2-6} = 0.23285$

Substitute F_{5-6}, F_{5-3}, F_{2-3} and F_{2-6} values in eqn ①, we get

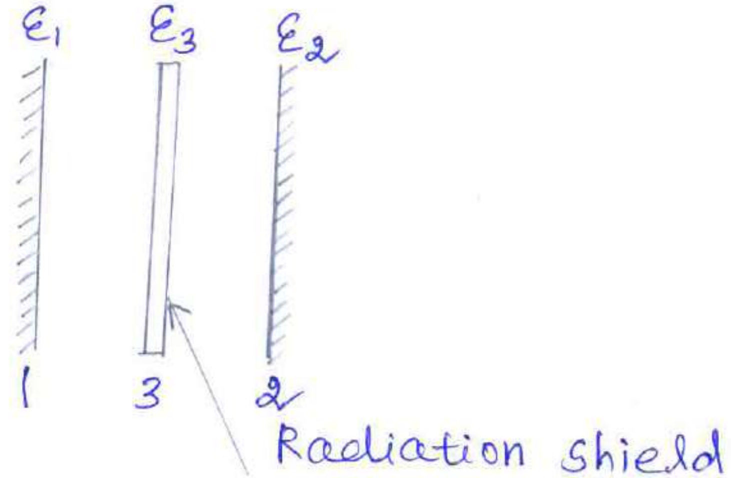
$$F_{1-4} = \frac{A_5}{A_1}\left[0.14930 - 0.11643\right] + \frac{A_2}{A_1}\left[0.20004 - 0.23285\right]$$

$$= \frac{2}{1}(0.03287) + \frac{1}{1}(-0.03281)$$

$F_{1-4} = 0.03293$

Radiation shield:

* Radiation shield constructed from low emissivity materials. It is used to reduce the net radiation transfer between two surfaces.



* Let us consider two parallel planes 1 and 2 each of area A at temperatures T_1 and T_2 respectively. A radiation shield is placed in between them as shown in figure.

* The net heat exchange between parallel plates without radiation shield is given by

$$Q_{12} = \frac{A\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} \quad \text{--- (1)}$$

Heat exchange between plates 1 and 3 is,

$$Q_{13} = \frac{A\sigma(T_1^4 - T_3^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_3} - 1} \quad \text{--- (2)}$$

Heat exchange between plates 3 and 2 is,

$$Q_{32} = \frac{A\sigma(T_3^4 - T_2^4)}{\frac{1}{\varepsilon_3} + \frac{1}{\varepsilon_2} - 1} \quad \text{--- (3)}$$

From eqn (2),

$$T_1^4 - T_3^4 = \frac{Q_{13}\left[\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_3} - 1\right]}{A\sigma}$$

$$T_3^4 = T_1^4 - \frac{Q_{13}\left[\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_3}-1\right]}{A\sigma}$$

Substitute T_3^4 value in eqn ③

$$Q_{32} = \frac{A\sigma\left\{\left[T_1^4 - \frac{Q_{13}\left(\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_3}-1\right)}{A\sigma}\right] - T_2^4\right\}}{\frac{1}{\varepsilon_3}+\frac{1}{\varepsilon_2}-1}$$

$$Q_{32}\left[\frac{1}{\varepsilon_3}+\frac{1}{\varepsilon_2}-1\right] = A\sigma T_1^4 - Q_{13}\left[\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_3}-1\right] - A\sigma T_2^4$$

$$Q_{32}\left[\frac{1}{\varepsilon_3}+\frac{1}{\varepsilon_2}-1\right] + Q_{13}\left[\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_3}-1\right] = A\sigma T_1^4 - A\sigma T_2^4$$

under equilibrium condition,

$$Q_{13} = Q_{32}$$

$$Q_{13}\left\{\left[\frac{1}{\varepsilon_3}+\frac{1}{\varepsilon_2}-1\right]+\left[\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_3}-1\right]\right\} = A\sigma(T_1^4 - T_2^4)$$

$$Q_{13} = \frac{A\sigma(T_1^4 - T_2^4)}{\left[\frac{1}{\varepsilon_3}+\frac{1}{\varepsilon_2}-1\right]+\left[\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_3}-1\right]} \quad \text{—} ④$$

Dividing the eqn ④ by eqn ①, we get

$$\frac{Q_{13}}{Q_{12}} = \frac{\left[\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_2}-1\right]}{\left[\frac{1}{\varepsilon_3}+\frac{1}{\varepsilon_2}-1\right]+\left[\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_3}-1\right]}$$

If $\varepsilon_1 = \varepsilon_2 = \varepsilon_3$

$$\frac{Q_{13}}{Q_{12}} = \frac{1}{2}$$

$$Q_{13} = \frac{1}{2} Q_{12} \quad (\text{or}) \quad Q_{32} = \frac{1}{2} Q_{12}$$

Formulae:

1. Heat exchange between two large parallel plate is given by

$$Q_{12} = \bar{\varepsilon} \sigma A (T_1^4 - T_2^4)$$

where $\bar{\varepsilon} = \dfrac{1}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$

2. Heat exchange between two large concentric cylinder (or) sphere

$$Q_{12} = \bar{\varepsilon} A_1 \sigma (T_1^4 - T_2^4)$$

For cylinder, $A = 2\pi r L$

Where $\bar{\varepsilon} = \dfrac{1}{\frac{1}{\varepsilon_1} + \frac{A_1}{A_2}\left(\frac{1}{\varepsilon_2} - 1\right)}$

For sphere, $A = 4\pi r^2$

3. Heat transfer with 'n' shield is given by

$$Q_{ln} = \frac{A\sigma (T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} + \frac{2n}{\varepsilon_s} - (n+1)}$$

where n – No. of shields

ε_s – Emissivity of shield

Q. 6

Given:-

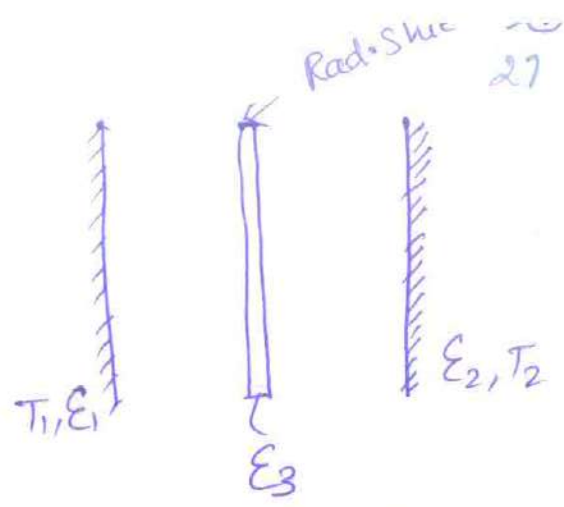
$\varepsilon_1 = 0.35$

$\varepsilon_2 = 0.85$

$T_1 = 1073^4$

$T_2 = 773^4$

$\varepsilon_3 = 0.04$



Solution:-

Case 1 Heat exchange without Rad-Shield

$$Q_{12} = \bar{\varepsilon}\,\sigma A\,(T_1^4 - T_2^4) \qquad \bar{\varepsilon} = \frac{1}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$

$$\frac{Q_{12}}{A} = 0.329 \times 5.67 \times 10^{-8} \times (1073^4 - 773^4) \qquad \bar{\varepsilon} = \frac{1}{\frac{1}{0.35} + \frac{1}{0.85} - 1}$$

$$\boxed{\frac{Q_{12}}{A} = 18067.017 \ \frac{W}{m^2}} \qquad \bar{\varepsilon} = \frac{1}{3.033}$$

$$\bar{\varepsilon} = 0.329$$

Case 2 Heat exchange with Rad-shield

$$Q_{13} = \bar{\varepsilon}\,\sigma A\,(T_1^4 - T_3^4) \quad \text{where, } \bar{\varepsilon} = \frac{1}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_3} - 1}$$

$$\boxed{Q_{13} = 0.0372\,\sigma A\,(1073^4 - T_3^4) \ \text{---} \ (1)} \qquad \bar{\varepsilon} = \frac{1}{\frac{1}{0.35} + \frac{1}{0.04} - 1}$$

$$Q_{32} = \bar{\varepsilon}\,\sigma A\,(T_3^4 - T_2^4) \qquad \bar{\varepsilon} = \frac{1}{26.857}$$

$$\text{where } \bar{\varepsilon} = \frac{1}{\frac{1}{\varepsilon_3} + \frac{1}{\varepsilon_2} - 1} \qquad \bar{\varepsilon} = 0.0372$$

$$\bar{\varepsilon} = \frac{1}{\frac{1}{0.04} + \frac{1}{0.85} - 1}$$

$$\bar{\varepsilon} = \frac{1}{25.176} = 0.0397$$

$$\boxed{\therefore Q_{32} = 0.0397\,\sigma A\,(T_3^4 - 773^4) \ \text{---} \ (2)}$$

under thermal equilibrium,

$$Q_{13} = Q_{32}$$

Substitute the values from eqn's ① and ②

$$0.0372\, \cancel{\sigma}\cancel{A}(1073^4 - T_3^4) = 0.0397\, \cancel{\sigma}\cancel{A}(T_3^4 - 773^4)$$

$$(0.0372 \times 1073^4) - 0.0372\, T_3^4 = 0.0397\, T_3^4 - (0.0397 \times 773^4)$$

$$(0.0372 \times 1073^4) + (0.0397 \times 773^4) = 0.0397\, T_3^4 + 0.0372\, T_3^4$$

$$(0.0372 \times 1073^4) + (0.0397 \times 773^4) = 0.0769\, T_3^4$$

$$T_3^4 = \frac{(0.0372 \times 1073^4) + (0.0397 \times 773^4)}{0.0769}$$

$$T_3 = \left[\frac{(0.0372 \times 1073^4) + (0.0397 \times 773^4)}{0.0769}\right]^{1/4}$$

$$\boxed{T_3 = 953.20\ K}$$

~~Format of the certification to be given by the Supervisor/Joint Supervisor (in all copies)~~

~~I certify that the thesis entitled ......~~
~~submitted for the Degree of Doctor of Philosophy by Mr./Ms. ......~~
~~is the bonafide record of research work carried out by him/her during the period from ...... to~~
~~...... under my/our guidance and supervision and that this research work has not formed the~~
~~basis for the award of any degree, diploma, associateship, fellowship or other similar titles in this~~
~~University or any other University or institution.~~

~~Date:~~
~~Place:~~ ~~Signature of the Supervisor with seal~~

~~Date:~~
~~Place:~~ ~~Signature of the Joint Supervisor with seal~~

① ⇒ $\frac{Q_{13}}{A} = 0.0372 \times 5.67 \times 10^{-8} (1073^4 - 953.20^4)$

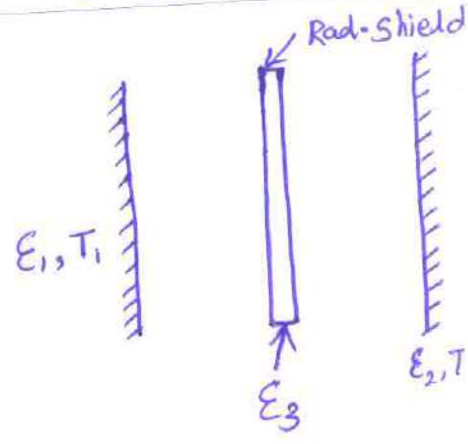
$$\boxed{\frac{Q_{13}}{A} = 1054.66 \frac{W}{m^2}}$$

$$\% \text{ reduction} = \frac{Q_{12} - Q_{13}}{Q_{12}}$$

$$= \frac{18067.017 - 1054.66}{18067.017} = \boxed{94.16\%}$$

---

e. 7 Given:-



$\varepsilon_1 = 0.8$

$\varepsilon_2 = 0.5$

$T_1 = 1000K$

$T_2 = 500K$

$\varepsilon_3 = 0.1$

Solution:- Case1:- Heat exchange without Rad. shield

$Q_{12} = \bar{\varepsilon} \sigma A (T_1^4 - T_2^4)$ where $\bar{\varepsilon} = \frac{1}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$

$\frac{Q_{12}}{A} = 0.444 \times 5.67 \times 10^{-8} \times (1000^4 - 500^4)$

$\bar{\varepsilon} = \frac{1}{\frac{1}{0.8} + \frac{1}{0.5} - 1}$

$\frac{Q_{12}}{A} = \underline{\underline{23,601.375 \ W/m^2}}$

$\bar{\varepsilon} = 0.444$

Case2: Heat exchange with Rad. shield

$Q_{13} = \bar{\varepsilon} \sigma A (T_1^4 - T_3^4)$ where $\bar{\varepsilon} = \frac{1}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_3} - 1}$

$\frac{Q_{13}}{A} = 0.0975 \sigma A (1000^4 - T_3^4)$ — ①

$\bar{\varepsilon} = \frac{1}{\frac{1}{0.8} + \frac{1}{0.1} - 1}$

$Q_{32} = \bar{\varepsilon} \sigma A (T_3^4 - T_2^4)$

$\bar{\varepsilon} = 0.0975$

where $\bar{\varepsilon} = \frac{1}{\frac{1}{\varepsilon_3} + \frac{1}{\varepsilon_2} - 1} = \frac{1}{\frac{1}{0.1} + \frac{1}{0.5} - 1} = 0.0909$

$Q \ \ = 0.0909 \sigma A (T_3^4 - T_2^4)$ — ②

Under equilibrium conditions,

$$Q_{13} = Q_{32}$$, substitute the values from equations ① and ②

$$0.0975\,\cancel{\sigma}\,\cancel{A}\,(1000^4 - T_3^4) = 0.0909\,\cancel{\sigma}\,\cancel{A}\,(T_3^4 - 500^4)$$

$$(0.0975 \times 1000^4) - 0.0975\,T_3^4 = 0.0909\,T_3^4 - (0.0909 \times 50$$

$$(0.0975 \times 1000^4) + (0.0909 \times 500^4) = 0.0909\,T_3^4 + 0.0975\,T_3^4$$

$$(0.0975 \times 1000^4) + (0.0909 \times 500^4) = 0.1884\,T_3^4$$

$$T_3^4 = \frac{(0.0975 \times 1000^4) + (0.0909 \times 500^4)}{0.1884}$$

$$T_3 = \left[\frac{(0.0975 \times 1000^4) + (0.0909 \times 500^4)}{0.1884}\right]^{1/4}$$

$$\boxed{T_3 = 860.26\ K}$$

$$① \Rightarrow \frac{Q_{13}}{A} = 0.0975 \times 5.67 \times 10^{-8} \times (1000^4 - 860.26^4)$$

$$\frac{Q_{13}}{A} = 2500.59\ W/m^2$$

$$\left.\begin{array}{l}\text{\% reduction in Radiation} \\ \text{Heat exchange}\end{array}\right\} = \frac{Q_{12} - Q_{13}}{Q_{12}}$$

$$= \frac{23{,}601.375 - 2500.59}{23{,}601.375}$$

$$\text{\% reduction} = 0.894 \times 100$$

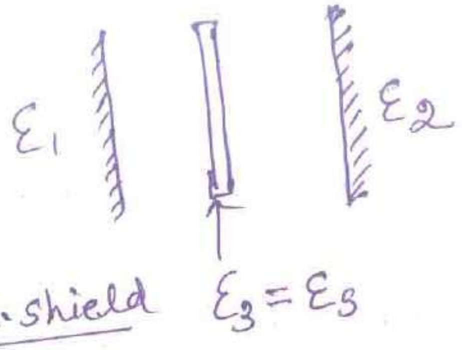
$$\boxed{= 89.4\%}$$

8 Given:

$\varepsilon_1 = 0.3$

$\varepsilon_2 = 0.8$

$\varepsilon_s = \varepsilon_3 = 0.04$



Solution:

Case 1: Heat exchange without Rad. shield

$$Q_{12} = \bar{\varepsilon}\sigma A (T_1^4 - T_2^4)$$

$$\bar{\varepsilon} = \frac{1}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} = \frac{1}{\frac{1}{0.3} + \frac{1}{0.8} - 1} = 0.279$$

$$\therefore Q_{12} = 0.279\, \sigma A (T_1^4 - T_2^4) \quad \text{—— (1)}$$

Case 2: Heat exchange with Rad. shield

$$Q_{13} = Q_{\text{with shield}} = \frac{A\sigma (T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} + \frac{2n}{\varepsilon_s} - (n+1)}$$

$$= \frac{A\sigma (T_1^4 - T_2^4)}{\frac{1}{0.3} + \frac{1}{0.8} + \frac{2\times 1}{0.04} - (1+1)} = \frac{A\sigma (T_1^4 - T_2^4)}{52.583}$$

$$Q_{\text{with shield}} = 0.019\, A\sigma (T_1^4 - T_2^4) \quad \text{—— (2)}$$

$$\left.\begin{array}{l}\text{\% reduction in}\\ \text{Radiation heat exchange}\end{array}\right\} = \frac{Q_{\text{without shield}} - Q_{\text{with shield}}}{Q_{\text{without Rad. shield}}}$$

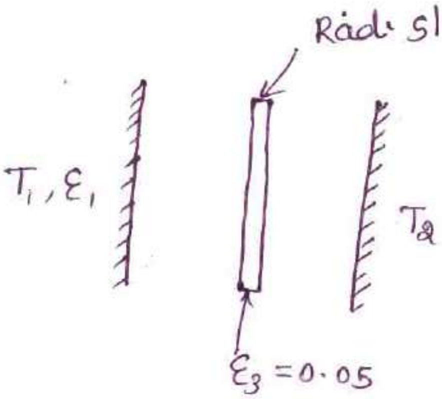
$$= \frac{0.279 - 0.019}{0.279} = 0.9318$$

$$\boxed{\text{\% reduction} = 93.18\%}$$

Given:-

Emissivity, $\varepsilon_1 = 0.8$

Emissivity, $\varepsilon_2 = 0.6$



Temp. for plate 1, $T_1 = 560°C + 273$

$T_1 = 833\ K$

Temp. for plate 2, $T_2 = 300°C + 273$

$T_2 = 573\ K$

Emissivity for Rad. shield. $\varepsilon_3 = 0.05$

Solution:-

Case 1: Heat exchange without radiation shield

$Q_{12} = \bar{\varepsilon}\sigma A(T_1^4 - T_2^4)$ where $\bar{\varepsilon} = \dfrac{1}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} = \dfrac{1}{\frac{1}{0.8} + \frac{1}{0.6} - 1}$

$\dfrac{Q_{12}}{A} = 0.5217 \times 5.67 \times 10^{-8} \times (833^4 - 573^4)$ $\quad \bar{\varepsilon} = 0.5217$

$\dfrac{Q_{12}}{A} = 11{,}053.66\ W/m^2$

Case 2:- Heat exchange with radiation shield

$Q_{13} = \bar{\varepsilon}\sigma A(T_1^4 - T_3^4)$ where $\bar{\varepsilon} = \dfrac{1}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_3} - 1} = \dfrac{1}{\frac{1}{0.8} + \frac{1}{0.05} - 1}$

$Q_{13} = 0.0493\,\sigma A(833^4 - T_3^4)$ — ① $\quad \bar{\varepsilon} = 0.0493$

$Q_{32} = \bar{\varepsilon}\sigma A(T_3^4 - T_2^4)$ where $\bar{\varepsilon} = \dfrac{1}{\frac{1}{\varepsilon_3} + \frac{1}{\varepsilon_2} - 1} = \dfrac{1}{\frac{1}{0.05} + \frac{1}{0.6} - 1}$

$Q_{32} = 0.0483\,\sigma A(T_3^4 - 573^4)$ — ②

Under Thermal equilibrium. $\quad \bar{\varepsilon} = 0.0483$

$Q_{13} = Q_{32}$

Substitute the values from equation ① and ②.

$0.0493\,\cancel{\sigma}\cancel{A}(833^4 - T_3^4) = 0.0483\,\cancel{\sigma}\cancel{A}(T_3^4 - 573^4)$

$(0.0493 \times 833^4) - 0.0493\,T_3^4 = 0.0483\,T_3^4 - (0.0483 \times 573^4)$

$$(0.0493 \times 833^4) + (0.0483 \times 573^4) = 0.0483\, T_3^4 + 0.0493\, T_3^4$$

$$(0.0493 \times 833^4) + (0.0483 \times 573^4) = 0.0976\, T_3^4$$

$$T_3^4 = \frac{(0.0493 \times 833^4) + (0.0483 \times 573^4)}{0.0976}$$

$$T_3 = \left[\frac{(0.0493 \times 833^4) + (0.0483 \times 573^4)}{0.0976}\right]^{1/4}$$

$$\underline{T_3 = 737.95\ K}$$

Substitute 'T_3' value in equation ①,

$$\frac{Q_{13}}{A} = 0.0493 \times 5.67 \times 10^{-8} \times (833^4 - 737.95^4)$$

$$\underline{\frac{Q_{13}}{A} = 516.92\ W/m^2}$$

$$\left.\begin{array}{l}\%\ \text{reduction in Radiation} \\ \text{Heat exchange}\end{array}\right\} = \frac{Q_{12} - Q_{13}}{Q_{12}}$$

$$= \frac{11053.66 - 516.92}{11053.66} = 0.9532$$

$$\underline{= 95.32\ \%}$$

Gas radiation (or) Radiation through Gases

* Many gases such as N_2, O_2, H_2 etc., not emit or absorb the thermal radiation.
* Some gases such as CO_2, CO, H_2O, SO_2, etc emit and absorb signifigant amount of radiant energy.
* Radiation from CO_2 and H_2O are the most common absorbing gases in atmosphere and industrial furnace.

Meanbeam length:-

$$L_m = 3.6 \times \frac{V}{A}$$

V - Volume of gas

A - Surface area of gas

10. A furnace of $25\ m^2$ area and $12\ m^3$ volume is maintained at a temp. of $925°C$ over its entire volume. The total pressure of the combustion gases is 3 atm, the partial pressure of water vapour is 0.1 atm and that of CO_2 is 0.25 atm. Calculate the emissivity of the gaseous mixture

Given:

Area, $A = 25\,m^2$

Volume, $V = 12\,m^3$

Temperature, $T = 925 + 273 = 1198\,K$

Total Pressure, $P = 3$ atm

Partial pr. of water vapour, $P_{H_2O} = 0.1$ atm

Partial pr. of CO_2, $P_{CO_2} = 0.25$ atm

To find:

$\varepsilon_{mixture}$

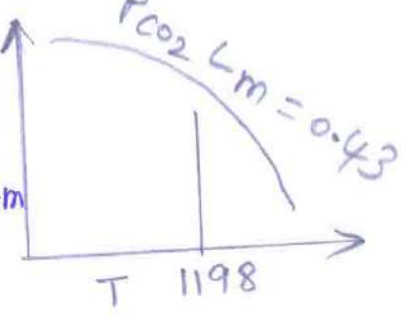
Solution:

Mean beam length for gaseous mixture

$L_m = 3.6 \times \frac{V}{A} = 3.6 \times \frac{12}{25} = 1.72\,m$

ε_{CO_2}

$P_{CO_2} \times L_m = 0.25 \times 1.72 = 0.43$ m-atm



From HMTDB, P 107

X-axis → Temp. curve → $P_{CO_2} \cdot L_m$

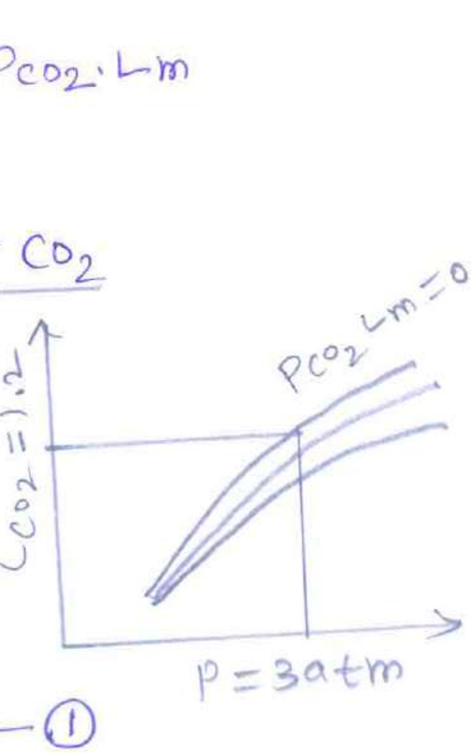
y-axis → $\varepsilon_{CO_2} = 0.15$

P 108, correction factor for CO_2

X-axis → Total pressure

curve → $P_{CO_2} L_m$

corresponding y-axis value

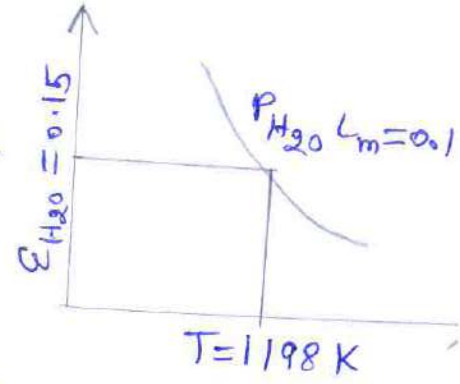


$C_{CO_2} = 1.2$

$0.15 \times 1.2 = 0.18$ — (1)

ε_{H_2O}

$P_{H_2O} \times L_m = 0.1 \times 1.72 = 0.172$



X-axis → Temp.

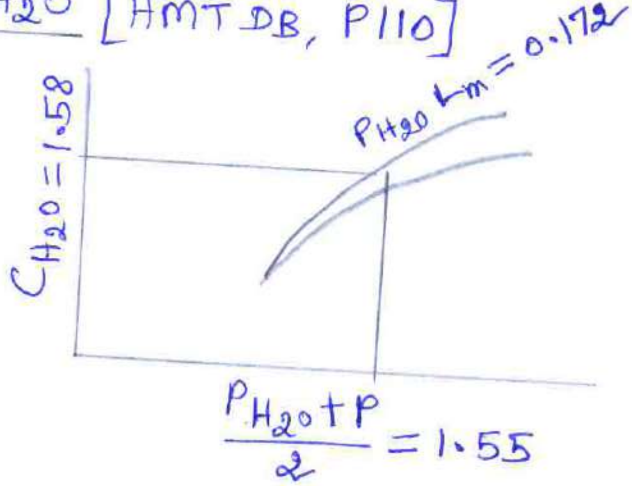
curve → $P_{H_2O} L_m$

Corresponding y-axis value is

$\varepsilon_{H_2O} = 0.15$

Correction factor for H_2O [HMTDB, P110]

$$\frac{P_{H_2O} + P}{2} = \frac{0.1 + 3}{2} = 1.55$$



$P_{H_2O} \times L_m = 0.172$

X-axis → $\frac{P_{H_2O} + P}{2}$

curve → $P_{H_2O} L_m$

Corresponding y-axis value is, $C\cancel{\varepsilon}_{H_2O} = 1.58$

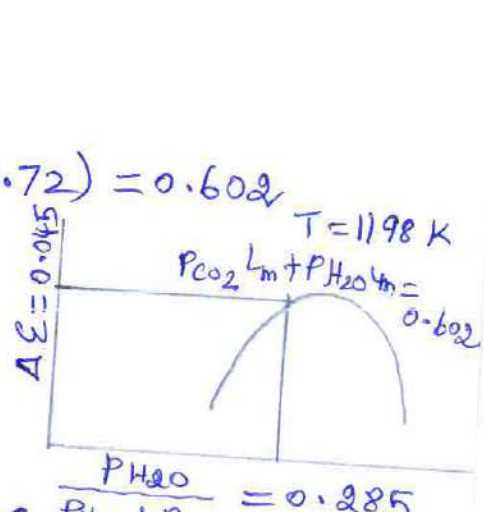
$\varepsilon_{H_2O} \times C_{H_2O} = 0.15 \times 1.58 = 0.237$ —— (2)

Correction factor for mixture of CO_2 & H_2O

$$\frac{P_{H_2O}}{P_{H_2O} + P_{CO_2}} = \frac{0.1}{0.1 + 0.25} = 0.285$$

$P_{CO_2} L_m + P_{H_2O} L_m = (0.25 \times 1.72) + (0.1 \times 1.72) = 0.602$

From HMTDB, P 111



X-axis → $\frac{P_{H_2O}}{P_{H_2O} + P_{CO_2}}$, Y axis → $\Delta\varepsilon = 0.045$

curve → $P_{CO_2} L_m + P_{H_2O} L_m = 0.602$

$\varepsilon_{mixture} = \varepsilon_{CO_2} \cdot C_{CO_2} + \varepsilon_{H_2O} \cdot C_{H_2O} - \Delta\varepsilon$

$\varepsilon_{mixture} = 0.18 + 0.27 - 0.045 = \boxed{0.372}$

UNIT - V MASS TRANSFER

Definition

* Whenever there is concentration difference of a physical quantity in a medium, nature tends to equalise things by forcing a flow from high to low concentration region.
* The process of transfer of mass as a result of the spieces concentration difference in a mixture is known as mass transfer.

Examples:-

1. Evaporation of petrol in carburettor
2. Humidification of air in cooling tower
3. Refrigeration by evaporation
4. Sugar added with milk

Modes of Mass transfer

1. Diffusion mass transfer
2. Convective mass transfer
3. Phase change mass transfer

1. Diffusion mass transfer

* The diffusion mass transfer is the transport of matter as a microscopic level. It occurs due to concentration difference only.

Example: Water evaporation from a sump

2. Convective mass transfer

* It occurs whenever the concentration of Particles at its surface differ from its concentration in a gas moving over the surface.

* It occurs due to concentration difference and velocity. Ex: Drying the clothes

3. Phase change mass transfer

* This is due to simultaneous effect of convection and diffusion mass transfer

Example: Boiling water in a open air

Mass concentration or Mass density

Air
N_2, O_2, H_2, He
CO_2, CO

* It is defined as mass of a component in a mixture per unit volume of a mixture.

$$\rho_A = \frac{\text{mass of a component}}{\text{Volume of the mixture}} = \frac{m_A}{V}, \ kg/m^3$$

Molar concentration or Molar density

* It is defined as number of moles of a component per unit volume of mixture.

$$C_A = \frac{\text{No. of moles of a component}}{\text{volume of the mixture}} = \frac{m_A/M_A}{V}$$

$$C_A = \frac{m_A}{V} \cdot \frac{1}{M_A} = \frac{\rho_A}{M_A}$$

1 mole of N_2 = 28 kg
1 mole of O_2 = 32 kg

M_A - Molecular weight of the component
m_A - mass of the component
V - Volume of mixture

Fick's law of diffusion

Molar flux of an element or diffusion rate of an element is directly proportional to concentration gradient.

$$\frac{M}{A} \propto \frac{dc}{dx}$$

$$\frac{M}{A} = D\,\frac{dc}{dx}$$

D - Diffusion coefficient, m^2/s

* considering a chamber having two chambers has Gas A and B in the chamber.
* The partition is removed, mass transfer by diffusion will be in the direction of lower concentration. After a sufficient long period equilibrium is achieved.

By Fick's law, $\frac{M_A}{A} \propto \frac{dc_A}{dx}$

$$\frac{M_A}{A} = -D_{AB}\,\frac{dc_A}{dx}$$

where $\frac{m_A}{A}$ - mass diffusion in unit area, Kg/sm^2

D_{AB} - Diffusion coefficient, m^2/s

$\frac{dc_A}{dx}$ - concentration gradient for component A

1. The composition of dry atmospheric air on a molar basis is 78.1% N_2, 20.9% O_2 and air rectify between constituents. Find the mass fraction of the constituents of air.

Given:-

Partial Pressure of O_2, $P_{O_2} = 20.9 \times 1\ bar = 20.9 \times 1 \times 10^5 N/m^2$

Partial pressure of N_2, $P_{N_2} = 78.1 \times 1 bar = 78.1 \times 1 \times 10^5 N/m^2$

Temperature, $T = 25°C + 273 = 298\ K$

Solution:-

Molar concentration, $C = \dfrac{P}{GT}$ $\quad [\because G = 8314\ J/Kg\text{-mole-}K]$

$$C_{O_2} = \frac{P_{O_2}}{GT} = \frac{20.9 \times 1 \times 10^5}{8314 \times 298}$$

$$C_{O_2} = 0.843\ Kg\text{-mole}/m^3$$

Molar concentration, $C_{N_2} = \dfrac{P_{N_2}}{GT} = \dfrac{78.1 \times 1 \times 10^5}{8314 \times 298}$

$$C_{N_2} = 3.152\ Kg\text{-mole}/m^3$$

W.K.T, Molar concentration $= \dfrac{\rho}{M} = \dfrac{\text{Density}}{\text{Molecular weight}}$

Density of oxygen, $\rho_{O_2} = C_{O_2} \times M$ $\quad$ [molecular weight of $O_2 = 32$]

$$\rho_{O_2} = 0.843 \times 32$$

$$\rho_{O_2} = 26.97\ Kg/m^3$$

$\dot{m}_{O_2} = \dfrac{\rho_{O_2}}{\rho} = 0.234$

Density of Nitrogen, $\rho_{N_2} = C_{N_2} \times M$ $\quad$ [$\because M$ for $N_2 = 28$]

$$\rho_{N_2} = 3.152 \times 28$$

$$\rho_{N_2} = 88.256\ Kg/m^3$$

$\dot{m}_{N_2} = \dfrac{\rho_{N_2}}{\rho} = 0.766$

Overall density, $\rho = \rho_{O_2} + \rho_{N_2} = 26.97 + 88.256$

$$\rho = 115.2\ Kg/m^3$$

2. The molecular weights of the two components A and B of a gas mixture are 24 and 48 respectively. The molecular weight of a gas mixture is found to be 30. If the mass concentration of the mixture is 1.2 kg/m^3, determine the following:

(i) Density of component A and B

(ii) Molar fractions

(iii) Mass fractions

(iv) Total pressure if the temperature of the mixture is 290K.

Given:-

molecular weight of component A, $M_A = 24$

" " " " B, $M_B = 48$

" " " gas mixture, $M = 30$

mass concentration, $\rho = 1.2 \text{ kg/m}^3$

Temperature, $T = 290\text{ K}$

To find:-

1. ρ_A and ρ_B
2. molar fractions, x_A & x_B
3. mass fractions, $\dot{m}_A$ & $\dot{m}_B$
4. Total pressure, P

Solution:-

molar concentration of mixture, $C = \frac{\rho}{M} = \frac{1.2}{30}$

$C = 0.04$

we know That, $\therefore C_A + C_B = 0.04$ — (1)

w.k.T. $\rho_A = M_A C_A = 24 C_A$

$\rho_A = 24 C_A$ — (2)

$\rho_B = M_B C_B = 48 C_B$

$\rho_B = 48 C_B$ — (3)

$\because \rho_A + \rho_B = \rho_{Total}$

$24\, C_A + 48\, C_B = \rho$ —— ④

Solving eqn ① & 4, we get

① × 24 ⇒ $24 C_A + 24 C_B = 24 \times 0.04$

④ ⇒ $24 C_A + 48 C_B = 1.2$

$C_A = 0.03$ kg mole/m^3

$C_B = 0.01$ kg mole/m^3

(i) Density, ρ_A & ρ_B

② ⇒ $\rho_A = 24\, C_A = 24 \times 0.03 = 0.72$ kg/m^3

③ ⇒ $\rho_B = 48\, C_B = 48 \times 0.01 = 0.48$ kg/m^3

(ii) x_A & x_B

$$x_A = \frac{C_A}{C} = \frac{0.03}{0.04} = 0.75$$

$$x_B = \frac{C_B}{C} = \frac{0.01}{0.04} = 0.25$$

(iii) $\dot{m}_A$ & $\dot{m}_B$

$$\dot{m}_A = \frac{\rho_A}{\rho} = \frac{0.72}{1.2} = 0.6$$

$$\dot{m}_B = \frac{\rho_B}{\rho} = \frac{0.48}{1.2} = 0.4$$

(iv) Total pressure @ 290 K

Gas law, $PV = mRT$

$$P = \frac{m}{V} RT \qquad \rho = \frac{m}{V}$$

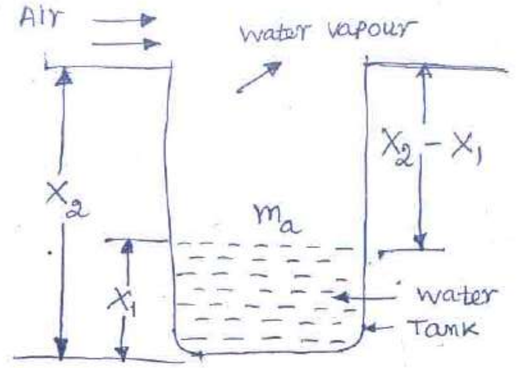
$$P = \rho RT. \qquad R = \frac{G}{M}$$

$$P = \rho \times \frac{G}{M} \times T = 1.2 \times \frac{8314}{30} \times 290$$

J/kg mole K

$P = 96442$ N/

Isothermal evaporation of water into Air



consider the isothermal evaporation of water from water surface and its diffusion through the stagnant air layer over it as shown in fig.

The free surface of water is exposed to air in the tank. Analysis of this type of mass diffusion, following assumptions are made.

1. The system is isothermal and total pressure remains constant.
2. System is in steady state condition.
3. There is slight air movement over the top of the tank to remove the water vapour which diffuses to that point.
4. Both the air and water vapour behave as ideal gases.

From Fick's law of diffusion,

Molar flux, $\frac{m_a}{A} = \frac{D_{ab}}{GT} \frac{P}{(x_2 - x_1)} \ln\left[\frac{P_{a2}}{P_{a1}}\right]$ or $\ln\left[\frac{P - P_{w2}}{P - P_{w1}}\right]$

where $\frac{m_a}{A}$ - molar flux - $\frac{kg\text{-}mole}{s\text{-}m^2}$

D_{ab} - Diffusion coefficient - m^2/s

G – Universal gas constant $= 8314 \ \frac{J}{Kg \cdot mole \cdot K}$

T – Temperature – K

P – Total Pressure in bar

P_{w_1} – partial pr. of water vapour at 1, N/m^2

P_{w_2} – partial pr. of dry air at 2, N/m^2

Pbm

3. An open pan 20 cm in diameter and 8 cm deep contains water at 25°C and is exposed to dry atmospheric air. If the rate of diffusion of water vapour is 8.54×10^{-4} Kg/h, estimate the diffusion coefficient of water in air.

Given:- Diameter, $d = 20 \text{ cm} = 0.2 \text{ m}$

Depth ~~Length~~ $(x_2 - x_1) = 8 \text{ cm} = 0.08 \text{ m}$

Temperature, $T = 25°C + 273 = 298 \text{ K}$

Diffusion rate or mass rate of water vapour $= 8.54 \times 10^{-4} \text{ Kg/h}$

$= \frac{8.54 \times 10^{-4}}{3600} = 2.37 \times 10^{-7}$ K

To find:- Diffusion co-efficient, D_{ab}

Solution:-

w.K.T.

Molar rate of water vapour $\frac{m_a}{A} = \frac{D_{ab}}{GT} \frac{P}{(x_2 - x_1)} \times \ln \left[\frac{P - P_{w_2}}{P - P_{w_1}}\right]$

$m_a = \frac{D_{ab} \times A}{GT} \times \frac{P}{(x_2 - x_1)} \ln \left[\frac{P - P_{w_2}}{P - P_{w_1}}\right]$

w.K.T,

Mass rate of water vapour $=$ Molar rate of water vapour $\times$ molecular weight of steam

$$2.37 \times 10^{-7} = \frac{D_{ab} \times A}{GT} \times \frac{P}{x_2 - x_1} \times 18.016 \times \ln \left[\frac{P - P_{w_2}}{P - P_{w_1}}\right]$$

Area, $A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.2)^2$

G = Universal gas const = 8314 J/kg-mole K

P = Total Pr. = 1 atm = 1.013 bar = 1.013×10^5 N/m²

P_{w_1} – Partial Pr. at bottom corresponding to Saturation temp. 25°C

<u>From Steam table</u> <u>P.2</u>

At 25°C, $P_{w_1} = 0.03166$ bar $= 0.03166 \times 10^5$ N/m²

P_{w_2} – Partial Pr. at top of the Pan.

Here no water vapour, so $P_{w_2} = 0$

$$① \Rightarrow 2.37 \times 10^{-7} = \frac{D_{ab} \times 0.0314}{8314 \times 298} \times \frac{1.013 \times 10^5}{0.08}$$

$$\times \ln\left[\frac{1.013 \times 10^5 - 0}{1.013 \times 10^5 - 0.03166 \times 10^5}\right] \times 18.016$$

$$D_{ab} = 2.58 \times 10^{-5} \text{ m}^2/\text{s} \qquad -[4.5 \times 10^{-5} \text{ m}^2/\text{s}]$$

[∵ molecular weight of steam is available in Page No. 184]

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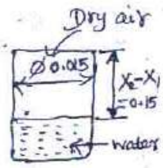
4. Estimate the diffusion rate of water vapour from the bottom of a test tube 1.5 cm diameter and 15 cm long into dry air at 25°C. Take $D = 0.256$ cm²/s

<u>Given</u>:- Diameter, $d = 1.5$ cm $= 0.015$ m



Depth, $(x_2 - x_1) = 15$ cm $= 0.15$ m

Temp. T $= 25°C + 273 = 298$ K

Diffusion co-efficient, $D_{ab} = 0.256$ cm²/s

$D_{ab} = 0.256 \times 10^{-4}$ m²/s

1 cm² = cm × cm
$= 10^{-2}$ m × 10^{-2} m
$= 10^{-4}$ m²

<u>To find</u>

Diffusion rate (or) mass rate of water vapour

<u>Solution</u>:- For Isothermal evaporation

$$\text{molar flux, } \frac{m_a}{A} = \frac{D_{ab}}{GT} \times \frac{P}{(x_2 - x_1)} \times \ln\left[\frac{P - P_{w_2}}{P - P_{w_1}}\right] \quad — ①$$

$$\text{Area, } A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.015)^2 = 1.76 \times 10^{-4} \text{ m}^2$$

Total pressure, $P = 1\,atm = 1.013\ bar = 1.013 \times 10^5\ N/m^2$

From steam table (P2)

Partial Pr. of water vapour at saturation temp 25°C $\left.\right\}$ $P_{w_1} = 0.03166\ bar$

$P_{w1} = 0.03166 \times 10^5\ N/m^2$

Partial Pr. of ~~dry air~~ top of the tube $\left.\right\}$ $P_{w_2} = 0.$

$$① \Rightarrow m_a = \frac{0.256 \times 10^{-4} \times 1.76 \times 10^{-4}}{8314 \times 298} \times \frac{1.013 \times 10^5}{0.15} \times \ln\left[\frac{1.013 \times 10^5 - 0}{1.013 \times 10^5 - 0.03166 \times 10^5}\right]$$

$$\text{molar rate} = 3.899 \times 10^{-11}\ \frac{kg\text{-mole}}{s}$$

W.K.T. From HMTDB P184, molecular weight of Steam = 18.016

$$\left.\begin{array}{l}\text{mass rate of}\\ \text{water vapour}\end{array}\right\} = \left\{\begin{array}{l}\text{molar rate of}\\ \text{water vapour}\end{array}\right\} \times \left\{\begin{array}{l}\text{Molecular weight}\\ \text{of steam}\end{array}\right\}$$

$$= 3.899 \times 10^{-11} \times 18.016$$

$$\left.\begin{array}{l}\text{mass rate or}\\ \text{Diffusion rate of}\\ \text{water vapour}\end{array}\right\} = 7.02 \times 10^{-10}\ kg/s$$

Types of Convective mass transfer:-

1. Free convective mass transfer
2. Forced Convective mass transfer

1. Free convective mass transfer

* If the fluid motion is produced due to change in density resulting from concentratio[n] gradients, the mode of mass transfer is free convective mass transfer.

2. Forced convective mass transfer

If the fluid motion is artificially created b[y] means of an external force (blower or fan), that type of mass transfer is forced convective

Schmidt, Sherwood and Lewis [illegible] Physical significance (Analogy between Heat transfer, mass transfer and momentum)

Schmidt Number:- It is defined as the ratio of the molecular diffusivity of momentum to the molecular diffusivity of mass.

$$Sc = \frac{\nu}{D_{ab}} \quad \begin{matrix} - m^2/s \\ - m^2/s \end{matrix}$$

Significance:- For problems involving both momentum and convection heat transfer.

Sherwood Number:- It is defined as the ratio of concentration gradients at the boundary.

$$Sh = \frac{h_m x}{D_{ab}}$$

h_m – mass transfer coefficient – m/s

D_{ab} – Diffusion coefficient – m²/s

x – Length – m

Significance:- Cases in which mass transfer is due to concentration difference.

Lewis Number:- It is the ratio between thermal diffusivity and diffusion coefficient

$$Le = \frac{\alpha}{D_{ab}}$$

α – thermal diffusivity,

D_{ab} – Diffusion coefficient, m²/s

Significance:- Cases in which both heat and mass transfer are present.

Formulae – Flat Plate

Reynolds Number, $Re = \frac{Ux}{\nu}$ or $Re = \frac{UL}{\nu}$

If $Re < 5 \times 10^5$, flow is laminar

If $Re > 5 \times 10^5$, flow is turbulent

For Laminar flow HMTDB P176

Local Sherwood Number, $Sh_x = 0.332 (Re_x)^{0.5} (Sc)^{0.333}$

Average Sherwood Number, $Sh = 0.664 (Re)^{0.5} (Sc)^{0.333}$

where, $Sc = \frac{\nu}{D_{ab}}$

$$Sh = \frac{h_m x}{D_{ab}}$$

For turbulent flow:- HMTDB P177

(i) Fully turbulent from leading edge

$$Sh = 0.0296 (Re)^{0.8} (Sc)^{0.333}$$

(ii) combined Laminar – Turbulent flow

$$Sh = [0.037\, Re^{0.8} - 871]\, Sc^{0.333}$$

$$Sh = \frac{h_m x}{D_{ab}}$$

5. Dry air at 27°C and 1 atm flows over a wet flat plate 50 cm long and velocity of 50 m/s. Calculate the mass transfer coefficient of water vapour in air at the end of the plate.

Given:- pressure, $P = 1\,atm = 1.013$ bar

Fluid temp., $T_\infty = 27°C$

Length, $x = L = 50\,cm = 50 \times 10^{-2}\,m$

Velocity, $U = 50\,m/s$

To find:- Mass transfer coefficient, (h_m)

Solution:- Properties of air at $27°C \simeq 30°C$

From HMTDB P34

$\nu = 16 \times 10^{-6}\,m^2/s$

W.K.T. $Re = \frac{UL}{\nu} = \frac{50 \times 50 \times 10^{-2}}{16 \times 10^{-6}} = 1.5625 \times 10^{6} > 5 \times 10^{5}$

$\therefore Re > 5 \times 10^{5}$, flow is turbulent

(But Flow is laminar upto $Re = 5 \times 10^{5}$)

For combined Laminar and Turbulent flow, (P177)

$$Sh = \left[0.037 (Re)^{0.8} - 871\right] Sc^{0.333} \quad \text{—} ①$$

$$S_c = \frac{\nu}{D_{ab}}$$

From HMT DB P181

(water + air) at 27°C ≃ 26°C

$$D_{ab} = 25.83 \times 10^{-6}\ m^2/s$$

$$\therefore Sc = \frac{16 \times 10^{-6}}{25.83 \times 10^{-6}} = 0.619$$

Substitute Re and Sc values in eqn ①

$$Sh = \left[0.037 \left(1.5625 \times 10^{6}\right)^{0.8} - 871\right] \times (0.619)^{0.333}$$

$$Sh = 2101.32$$

W.K.T. Sherwood Number, $Sh = \frac{h_m x}{D_{ab}}$

$$2101.32 = \frac{h_m \times 50 \times 10^{-2}}{25.83 \times 10^{-6}}$$

$$\therefore h_m = \frac{2101.32 \times 25.83 \times 10^{-6}}{50 \times 10^{-2}}$$

$$h_m = 0.108\ m/s$$

6. Air at 25°C flows over a tray full of water with a velocity of 2.8 m/s. The tray measures 30 cm along the flow direction and 40 cm wide. The Partial pressure of water present in the air is 0.007 bar. Calculate the evaporation rate of water if the temperature on the water surface is 15°C. Take diffusion coefficient is $4.2 \times 10^{-5}\ m^2/s$.

Given:

Fluid temp, $T_\infty = 25°c$

Velocity, $U = 2.8 \, m/s$

Length, $x = L = 30 \, cm = 0.3 \, m$

Wide, $W = 40 \, cm = 0.4 \, m$

Area, $A = L \times W = 0.3 \times 0.4 = 0.12 \, m^2$

Partial Pressure of water, $P_{w_1} = 0.007 \, bar = 0.007 \times 10^5 \, N/m^2$

Temp on water surface, $T_W = 15°c$

Diffusion coefficient, $D_{ab} = 4.2 \times 10^{-5} \, m^2/s$

To find:-

Evaporation rate of water, (m_w)

Solution: Film temp., $T_f = \frac{T_W + T_\infty}{2} = \frac{15+25}{2}$

$T_f = 20°c$

Properties of air at 20°c, From HMTDB P 34

$\nu = 15.06 \times 10^{-6} \, m^2/s$

w.k.t. $Re = \frac{UL}{\nu} = \frac{2.8 \times 0.3}{15.06 \times 10^{-6}} = 0.5577 \times 10^5 < 5 \times 10^5$

$\boxed{Re = 5.577 \times 10^4} < 5 \times 10^5$

$\because Re < 5 \times 10^5$, flow is laminar

For flat plate, Laminar flow, From HMTDB P176

$Sh = 0.664 (Re)^{0.5} (Sc)^{0.333}$ — (1)

$Sc = \frac{\nu}{D_{ab}} = \frac{15.06 \times 10^{-6}}{4.2 \times 10^{-5}} = \boxed{0.3585}$

Substitute Re, Sc values in eqn (1)

$Sh = 0.664 (5.577 \times 10^4)^{0.5} \times (0.3585)^{0.333}$

$Sh = 111.43$

we know that, $Sh = \frac{h_m x}{D_{ab}} \Rightarrow h_m = \frac{Sh \times D_{ab}}{x}$

$h_m = \frac{111.43 \times 4.2 \times 10^{-5}}{0.3}$

$h_m = 0.0156 \, m/s$

Evaporation rate of water, $m_w = h_{mp} \times A \left[P_{w_1} - P_{w_2}\right]$ —(2)

From Steam table P(1)

For water surface temp, $T_w = 15°C$

Saturation pr of water at 15°C, $P_{w_1} = 0.017 \text{ bar} = 0.017 \times 10^5 \text{ N/m}^2$

Mass transfer coefficient based on pressure, $h_{mp} = \dfrac{h_m}{R \times T_w \text{ in 'K'}}$

$$h_{mp} = \frac{0.0156}{287 \times 288} = 1.887 \times 10^{-7} \text{ m/s}$$ ($R = 287$ J/kg)

$T_w = 15 + 273 = 288$ K

Substitute the h_{mp}, A, P_{w_1} and P_{w_2} values in eqn (2),

Evaporation rate of water, $m_w = 1.887 \times 10^{-7} \times 0.12 \times \left[0.017 \times 10^5 - 0.007 \times 10^5\right]$

$$m_w = 2.264 \times 10^{-5} \text{ Kg/s}$$

Formulae – cylinder or Pipes (Internal flow)

1. $Re = \dfrac{UD}{\nu}$

If $Re < 2000$, flow is laminar

If $Re > 2000$, flow is turbulent

For laminar flow ($Re < 2000$)

Sherwood Number, $Sh = 3.66$

Also $Sh = \dfrac{h_m D}{D_{ab}}$

For turbulent flow ($Re > 2000$) HMTDB P177

Sherwood Number, $Sh = 0.023 (Re)^{0.83} (Sc)^{0.44}$

Also $Sh = \dfrac{h_m D}{D_{ab}}$

Schmidt No., $Sc = \dfrac{\nu}{D_{ab}}$

7. Air at 30°C and atmospheric pressure flows in a 12 mm diameter tube of 1 m length with a velocity of 2.5 m/s. The inside surface of the tube contains a deposit of naphthalene. Determine the average mass transfer coefficient. Take $D_{ab} = 0.62 \times 10^{-5}\ m^2/s$.

Given:-

Fluid temp., $T_\infty = 30°C$

Velocity, $U = 2.5\ m/s$

Dia., $D = 12\ mm = 0.012\ m$

Length, $x = 1\ m$

$D_{ab} = 0.62 \times 10^{-5}\ m^2/s$

To find:- Average mass transfer coefficient, h_m

Solution:- Properties of air at 30°C, From HMTDB P34

$\nu = 16 \times 10^{-6}\ m^2/s$

$$Re = \frac{UD}{\nu} = \frac{2.5 \times 0.012}{16 \times 10^{-6}} = 1.875 \times 10^3 < 2000$$

$\therefore$ Flow is laminar.

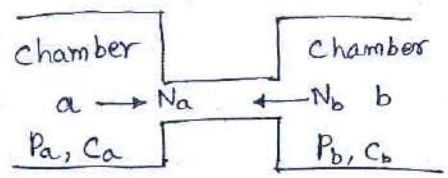
$\therefore\ Sh = 3.66$

Also $$Sh = \frac{h_m D}{D_{ab}} \Rightarrow h_m = \frac{Sh \times D_{ab}}{D}$$

$$h_m = \frac{3.66 \times 0.62 \times 10^{-5}}{0.012}$$

$$h_m = 1.89 \times 10^{-3}\ m/s$$

Steady state (or) Equimolar counter diffusion
molecular diffusion



$N_a = \frac{m_a}{A}$

Consider chamber 'a and b is connected by a Passage.

N_a and N_b are molar [diffusion rates of] flux component a and b.

* Equimolar diffusion is defined as each molecules of component a is replaced by each molecules of " " b.

Total pressure, $P = P_a + P_b$

Differenciate w.r.to x

$$\frac{dP}{dx} = \frac{dP_a}{dx} + \frac{dP_b}{dx}$$

Total pr is const. under steady state.

$$\therefore \frac{dP}{dx} = \frac{dP_a}{dx} + \frac{dP_b}{dx} = 0$$

$$\frac{dP_a}{dx} = -\frac{dP_b}{dx} \quad \text{——} (1)$$

Total molar flux is zero under steady state

$$N_a + N_b = 0$$

$$N_a = -N_b$$

From Fick's law,

$$N_a = \frac{\cancel{m_a}}{A} = -D_{ab}\frac{A}{GT}\frac{dP_a}{dx}$$

$$N_b = -D_{ba}\frac{A}{GT}\frac{dP_b}{dx}$$

$$-\cancel{D_{ab}}\frac{\cancel{A}}{GT}\frac{dP_a}{dx} = \cancel{D_{ba}}\frac{\cancel{A}}{\cancel{GT}}\frac{dP_b}{dx}$$

$$-\frac{dP_a}{dx} = \frac{dP_b}{dx}$$

Substitute $\frac{dP_b}{dx} = -\frac{dP_a}{dx}$ in eqn ②

$$-D_{ab}\cancel{\frac{A}{GT}}\cancel{\frac{dP_a}{dx}} = -D_{ba}\cancel{\frac{A}{GT}}\cancel{\frac{dP_a}{dx}}$$

$$D_{ab} = D_{ba} = D$$

$$\therefore \quad N_a = m_a = -D\frac{A}{GT}\frac{dP_a}{dx}$$

$$N_a = \frac{m_a}{A} = \frac{-D}{GT}\cdot\frac{dP_a}{dx}$$

$\int$ing, we get

$$N_a = \frac{m_a}{A} = -\frac{D}{GT}\int_1^2 \frac{dP_a}{dx}$$

$$N_a = -\frac{D}{GT}\left[\frac{P_{a_2} - P_{a_1}}{x_2 - x_1}\right]$$

$$N_a = \frac{D}{GT}\left[\frac{P_{a_1} - P_{a_2}}{x_2 - x_1}\right]$$

Similarly,

$$N_b = \frac{m_b}{A} = \frac{D}{GT}\left[\frac{P_{b_1} - P_{b_2}}{x_2 - x_1}\right]$$

where

$\frac{m_a}{A}$ – molar flux

D – Diffusion coef

G – Universal Gas const.

A – Area

P_{a_1} – Partial pr at 1

P_{a_2} – " " 2

T – Temp in K